Physics 740: Spring 2007: 05/07/07

P740X20.tex; In class exam 2.

1. For an incompressible fluid $\nabla \cdot \mathbf{v} \neq 0$? Y ——, **NO!.**, for $\rho =$ constant the continuity equation reduces to $\nabla \cdot \mathbf{v} = 0$.

2. When H.2 is written in the form

$$\frac{P - P_0}{V - V_0} = \rho_0^2 (D - u_0)^2 \tag{1}$$

the units of V_0 are $\mathbf{L^3/M}$, e.g., $[P/\rho] = [E]/M$ and $[\rho D^2] = [E]/L^3$ or ... $[\cdots]$=units of $\cdots$.

3. In the equation $\delta^2 = 2D_\eta/\omega$ the quantity δ is called **viscous penetration depth** and D_η is given in terms of to η by the equation $D_\eta =$ η/ρ .

4. The dispersion relation

   (a) $\omega^2 = gk$ describes **deep water waves**.

   (b) $\omega^2 = \gamma k^3$ describes **waves driven by surface tension** .

   (c) $\omega^2 = gh_0(k^2 - k^4 h_0^2/3 + \cdots)$ describes **shallow water waves at finite** kh_0.

5. In question 4 which waves have dispersion (a) **Yes**, (b) **Yes**, (c) **Yes**, $\omega \neq ck$.

6. In question 4 which waves have attenuation (a) **No**, (b) **No**, (c) **No**, ω is not complex.

7. For a shallow water wave $kh_0 \ll 1$? **Yes**, N—— .

8. If

$$f(\omega) = \alpha\omega^2 + i\beta\omega, \tag{2}$$
$$f(-\omega) = \alpha\omega^2 - i\beta\omega, \tag{3}$$

then $f(t)$ (to answer (a) and (b) you need ω vs k)

   (a) has attenuation Y ——, **No**

   (b) has dispersion Y ——, **No**

(c) is real **Yes**, N—— ? ($f(-\omega) = f(\omega)^*$ for real)

9. For the 3 fluids shown in Fig. 1 what are the contact angles? (a) $\theta = \mathbf{135^o}$, (b) $\theta = \mathbf{60^o}$, (c) $\theta = \mathbf{90^o}$.

10. The equation

$$\frac{\partial^2 \delta h}{\partial \tau^2} = \frac{\partial}{\partial z}\left(\left[1 - z^2\right]\frac{\partial \delta h}{\partial z}\right), \tag{4}$$

comes from the equation

$$\frac{\partial^2 \delta h}{\partial t^2} = g\frac{\partial}{\partial x}\left(h_0\left[1 - \frac{x^2}{a^2}\right]\frac{\partial \delta h}{\partial x}\right), \tag{5}$$

upon making the changes of variable **z=x/a** and $\tau = \omega$ **t**, where ω^2=**g**h_0/a^2.

11. The equations

$$\dot{u} = \frac{\partial^2 u}{\partial x^2}, \tag{6}$$
$$\ddot{u} = \frac{\partial^2 u}{\partial x^2}, \tag{7}$$
$$\ddot{u} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u^2}{\partial x^2}, \tag{8}$$
$$\dot{x}^2 = -x^2, \tag{9}$$
$$\dot{x} = -sinx, \tag{10}$$

are

(a) (7) **linear**

(b) (8) **linear**

(c) (9) **nonlinear**

(d) (10) **linear**

(e) (11) **nonlinear**

12. An aluminum plate atop a fluid is driven at frequency 0.10 Hz. The velocity field in the fluid is a shown in Fig. 2. What is D_η for the fluid? $D_\eta = \pi\delta^2/T_\omega \approx 30$, where $\delta \approx 10$ from the figure. The needed equation is in problem 3.

13. The units of pressure are **dyne/cm^2** ; the units of viscosity are **gm/cm-sec** , the units of γ_{SL} are **gm/sec^2** (cgs).

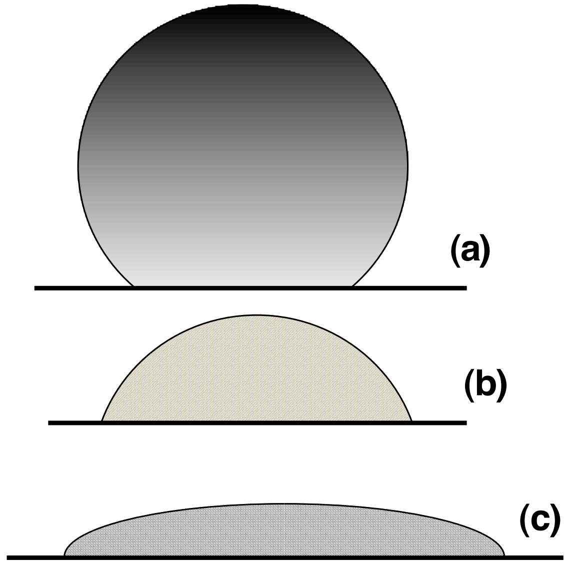

FIG. 1: three fluids on a surface.

14. In H.1,

$$D = \frac{F_0 - F}{\rho_0 - \rho}, \tag{11}$$

F, called **flux** (or current), is given by $F = \rho u$?

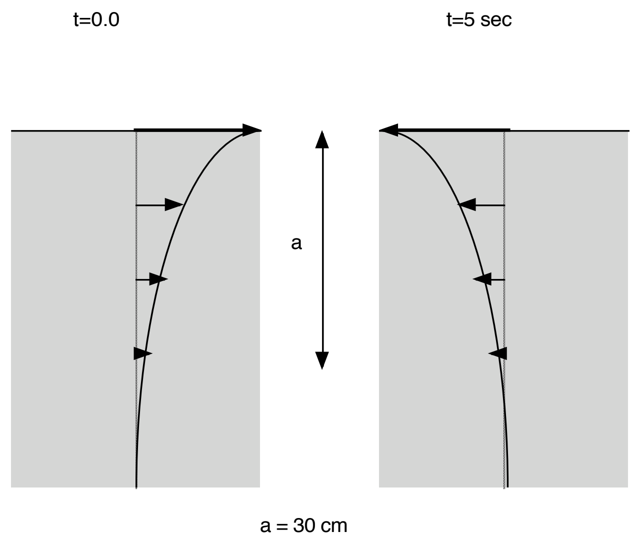

FIG. 2: Fluid motion at times separated by 1/2 period.