**Novel model of viscoelasticity for ionic electroactive polymer actuators**

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**Abstract** A non-traditional approach to the elements of the traditional viscoelastic schemes – spring and damper – results with a qualitatively new model of viscoelasticity. Combining series of short entities containing the lumped electrical circuit, and the bending moment generated according to the novel viscoelastic scheme results with a distributed model of the ionic electroactive polymer actuators. This model takes into account the superposition principle and describes among other features the back-relaxation of the actuators after their stimulated bending forward. The experiments carried out with three actuators of different electrode and membrane materials show excellent accordance with the model.

**1. Introduction**

Already since the very first reports concerning the bending ionic electroactive polymer (iEAP) actuators, the researchers have described the back-relaxation effect – an actuator excited with DC voltage, instead of holding its bent state, relaxes slowly back towards its initial shape [1-4]. This behavior is commonly treated as a shortcoming of iEAP actuators, decreasing their ability to deliver a constant peak force to target, limiting their frequency range, and hindering their exact control [5, 6]. There have been several attempts to slower the rate of back-relaxation by choosing an opportune combination of separator, electrodes, mobile ions [7-10] or driving signals [11], but the issue is still topical.

Supposedly, the phenomenon of back-relaxation is studied almost only in the case of „old-type“ ionic polymer-metal composites (IPMC). This is a wet ion exchange polymer membrane covered with thin metal electrodes on both sides [1]. For typical IPMC with platinum electrodes, when a voltage is applied across the thickness direction, the electromechanical transduction occurs due to the motion of the cations along with the water molecules. As a result, one face of IPMC withers and shrinks, while the opposite face protracts due to the swelling of water and ions. This generates warping of the whole laminate. The resulting fast bending motion towards the cathode is followed by a slow relaxation towards the anode, while IPMC as a whole still remains bent. If now the input of this actuator is shorted, the actuator performs a sharp movement towards its initial state, but will usually outdistance. The steps of this process are depicted in figure 1. The speed of back-relaxation is reported being dependent on the type of polymer membrane, solvent, cations, etc. [12]. It is commonly believed that the slow relaxation happens due to the water diffusion out of the strained polymer matrix [1]. Most of the proposed electromechanical models describing the behavior of IPMC, just neglect the phenomenon of back-relaxation. The few exceptions are presented by Bao et al. [13], proposing the model of IPMC without and with back-relaxation by adding a separate relaxation time constant, and the scalable model for the complete actuation response of IPMC of McDaid et al. [14].

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| (a) | (b) | (c) | (d) |

**Figure 1**. Bending response of IPMC to the input signal given above. (a) – initial relaxed state; (b) – forward bending response to a constant input voltage; (c) – back-relaxation with voltage still applied; (d) – opposite bending due to shorted input. The watch serves for time monitoring.

In recent years the range of iEAP materials has been significantly extended by means of the materials used for electrodes, separator membrane, as well as the electrolyte. At first glance, all ionic EAPs seem similar in construction – two conducting electrodes separated by a polymer membrane, containing moving ions – but their actuation mechanisms can be significantly different. There haven’t appeared any definitive models or at least reports yet, describing the phenomenon of back-relaxation of iEAP actuators with electrodes based on carbon or conductive polymers.

Experimenting with the iEAP actuators with carbon electrodes, we have noticed that similarly to the water-containing IPMC, virtually all of them exhibit back-relaxation, at least to some extent. It appears that by means of their transient spatial actuation and transient moment of force, the long-term behavior of all iEAP actuators is just similar. The back-relaxation is present regardless of the absence of the fluent liquid, and regardless of the membrane or electrode material.

The bending-relaxing behavior of iEAP actuators resembles viscoelasticity, but seems working partly conforming, partly contra to the electrical excitation. For this behavior, the constitutive models of viscoelasticity, consisting of linear combinations of springs and dampers, are inappropriate. There exist numerous models describing the response of viscoelastic materials and devices in different loading conditions, e.g. Maxwell model, Kelvin-Voigt model, or combinations of both of them [15]. All the models describe only the response of the material to the external load. Alas, neither of them involves any element causing the actual behavior of the iEAP actuators – lapsing against the load. The cause of this failure is the pattern of the so-called canonic models of viscoelasticity [15], where the load is external, and is applied to the whole system. Applying the exciter between the spring and damper instead of being applied to both in parallel will result with a completely new scheme of viscoelasticity, describing the effect of leaning against the exciter naturally.

The objective of the current paper is to describe the time-dependent correspondence between the transient input voltage and the shape of the iEAP actuators, taking into account the back-relaxation. We ground to the scalable distributed model of IPMC, presented in [16], and extend it to different iEAP materials. This approach describes the commonly uneven propagation of voltage along the iEAP material, and associates the voltage and charge carried over with the uneven flexure of the actuator. The experiments described in [16] assume that the charge-flexure relation is simply linear, and leave off just before the phenomenon of back-relaxation becomes noticeable. The current paper associates our novel model of viscoelasticity with the electrical model of IPMC, introduced in [16]. The resulting electromechanical model of iEAP allows describing the shape or blocking force of the actuators over a long time. Due to the accordance to the Boltzmann superposition principle – each loading step makes independent contribution to the total loading history [17] it works with any arbitrary transient input signal.

This paper is organized as follows: Firstly, in section 2 we show that the innovatory approach to an assembly of springs and dampers will result with PDE-s describing the truthful bending of the actuators forwards and backwards under a constant exciter. In section 3 we demonstrate the capabilities of the proposed model of viscoelasticity with the response of a lumped model of a fictional IPMC. The thorough explanation of the characterization of the mechanics of iEAP actuators given in section 4 is inevitable for understanding the distributed electromechanical model introduced in section 5. The verification of the model along with the examples based on three entirely different iEAP materials is presented in section 6. The discussion in section 7 draws conclusions and disputes about the issues left open.

**2. The model of viscoelasticity with internal strain**

As mentioned hereinabove, in the canonic models of viscoelasticity the load is external, and is applied to the whole combination of springs and dampers. This configuration does not involve even any possibility to suppress the exciter. The situation changes completely, when the exciting factor is applied between the spring and damper, resembling some factor, expanding-contracting between the elastic threads of the polymer molecules network. The viscoelastic scheme is depicted in figure 2. It consists of a damper in series with our new element - internal strain -, both in parallel with a lone spring. The internal strain is a vector of variation of the length of a non-elastic element. Hereinafter we associate it with the transient electrical charge of the iEAP material, so for clarity we label the strain element with Q and portray with Q between arrows. When the internal strain of the element Q is applied, the spring reacts momently, while the damper follows with time, tending towards the unstressed state of the spring. In brief, the input of our viscoelastic element is the strain, while the output is the strain of the spring.



**Figure 2**. The model of viscoelasticity with internal strain.

The behavior of this viscoelastic model in different loading conditions is depicted in figure 3. Initially, the spring and the damper are in their unstressed states: (figure 3(a)). The positive strain of Q draws the spring out (figure 3(b)) while the damper releases in time. This process continues regardless of the applied strain Q until the stress of spring is vanished (figure 3(c)). The removal of the strain Q results with the compression of the spring (figure 3(d)), followed by gradual dragging of the damper, until the stress of the spring is again zero (figure 3(e)). The negative strain of Q is depicted as its shortening (figure 3(f)) and compression of the spring. Similarly, the spring draws the damper longer until reaching the unstressed state (figure 3(g)). As seen in figure 3, this system strives to gain its unstressed state with any strain Q.

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| (a) | (b) | (c) | (d) | (e) | (f) | (g) |

**Figure 3**. Response of the viscoelastic scheme in different loading conditions.

The posed model is defined with the following four relations:

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where and are modulus of elasticity of the spring and viscosity of the damper respectively.

We see two equivalent solutions to this model: with respect to the input strain and with respect to the speed of strain , here we present both of them.

The relations (1)-(4) can be arranged into differential equation for the strain of damper as

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The solution for is found by considering (2) and solving (5) for with initial condition

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A different form of the equation is derived by combining (1)-(4) for the strain of the spring directly:

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Solving (7) with yields

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This form might be useful if the derivative of is known. By introducing the rate of relaxation and expressing the input strain in terms of stress both given forms are simplified to (9) and (10).

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**3. Lumped model of iEAP with relaxation**

In order to demonstrate the capabilities of the proposed model of viscoelasticity and its correspondence to the superposition principle, we use a simple lumped model of IPMC presented e.g. in [13]. That concept couples the input voltage with transferred charge and the tip displacement of an IPMC actuator. It gives reasonable results in case of small displacements only but serves well to explain the complicated approach presented in the next sections of this paper. Here we just demonstrate the transient response of the model to a rectangular input signal without attempting to validate it with any real actuator.

The lumped model of IPMC is presented in figure 4(a) while the electrical signals as well as strains are depicted in figure 4(b). When voltage is switched on, the charge curve of the capacitance is exponential with respect to time, and the correspondence between the strain (or tip displacement) and charge is proportional: . The diagram given in figure 4(b) depicts the rectangular voltage ; the charge of the capacitance ; and two, according to (9) numerically modeled transient strain behaviors with different parameters and convenient values of the coefficient to fit the graphs into the picture.

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| (a) | (b) |
| **Figure 4**. Equivalent circuit (a) and behaviour (b) of the lumped model of IPMC. | |

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Supposing that initially the system has been shorted for long time, the initial charge of is zero: . When the voltage is turned on, the strain curve initially follows the charge curve , turns apart when the integral part of (9) grows, then relaxes slowly with the speed determined by . When the voltage is switched off, the sharp bending towards the initial state keeps the curve of charge followed by back-relaxation, but according to the superposition principle starts from the momentary values of strain and charge. Similar back-relaxing behavior of IPMC is described in numerous papers, e.g. in [4, 13, 18].

**4. Distributed bending moment of the actuator**

Describing the bending iEAP actuators, it is convenient to observe only its short segment, for instance its tip. So far, lots of the publications follow this pattern and model the tip displacement in Cartesian coordinates, the blocking force of the tip, etc. [19-21]. These approaches assume that the shape of the actuator is characterized with a single radius of curvature, and the strain of the actuator reflects directly the tip displacement. This unsophisticated concept can be true in the case of small unobstructed displacements only. As soon as the movement of the actuator is restricted, for instance by the gauge measuring the blocking force, its shape will deviate from circular, and the whole concept fails. The vectorial interpretation presented in this section is universal and serves as the first step in modeling the distributed bending moment of the actuators independently of their actual shape.

*4.1. Characterization of the shape of the bending actuator*

The vectorial interpretation of the shape of the actuator expresses its bending with respect to the distance from the input contacts along the sample. The curved line representing the shape of the actuator is divided into vectors of equal lengths as depicted in figure 5, assuming that within every vector the curvature is constant. The shape of the actuator is characterized by a number of angles. Each next angle along the length of the actuator is relative to the previous one, while the angles of the first column may be arbitrary.

The image of the actuator is recorded by a camera and may be processed by some convenient image processing software or even manually with the ruler and protractor. A proper lighting and exposure greatly simplifies the procedure of placing the vectors. When the direction of the camera is set transverse to the actuator and the stage is illuminated from the background, the image of the actuator consists of a single contrast curved line. Applying the procedure for sequential images gives a 3-dimensional evolution of curvature along the actuator in time. As an example, the graph given in figure 6 depicts the actuating-relaxing behavior of the actuator presented in figure 1.

The length and total number of the vectors is a trade-off between the accuracy of the result and the resolution of the camera. It is self-evident that more vectors represent a complicated shape better. However, the relative error of detecting the angle depends on the discretization of the recorded image. Hence, it is important to provide sufficient resolution of the pixelated actuator to minimize image processing errors.

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| **Figure 5**. Vector representation of an actuator. | **Figure 6**. Transient behavior of an IPMC actuator. |

*4.2. Characterization of the bending moment of the bending actuator*

In cantilever mode – fixed end attached to contacts – the bending of the actuator is determined by the summarized moment of several components caused by the effects of osmotic, electrostatic and elastic stresses of the sample. When the sample is mounted edgewise, the component caused by the weight of the actuator in the particular environment in the plane of bending can be disregarded. In the scope of the current paper, the mass and inertia of the actuator are neglected and each time step is treated as an independent static problem. According to the Euler-Bernoulli law the bending moment along a cantilever beam is proportional to the local curvature:

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where is the modulus of elasticity and is the second moment of area.

When the cross-section of samples is rectangular, its second moment of area is expressed as

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where and are width and height of the cross-section of the sample respectively.

The described methodic does not pose any restrictions to the shape of the actuator, it works as well when the second moment of inertia is not constant along the sample e.g. the sample is tapered, or its cross-section differs from rectangular. In our case the summarized bending moment of the time-dependent behavior of the sample presented in figure 1 reflects exactly its bending depicted in figure 6, yet in a different vertical scale.

**5. Viscoelastic electromechanical model of iEAP actuators**

*5.1. Previous work*

The concept of the distributed electromechanical model of iEAP actuators is introduced in [16]. This approach declares that the bending of an IPMC actuator at any time and any point is caused by the charge carried over between the electrodes at that point by that time. The amount of charge is determined by the inhomogeneous transitory voltage that in turn is uneven due the electrical resistance of the electrodes. The electrical constituent of that model depicted in figure 7 resembles a sophisticated RC transmission line. The conductivity of the electrodes of the iEAP material is represented by a series of resistances of the opposite electrodes and , while the capacitance , and the loss parameters form the impedance of the material.



**Figure 7**. iEAP resembling a lossy RC transmission line.

Although the paper [16] emphasizes that the flexure of the IPMC actuator is non-uniform in spatial and time domain and that it is scalable by means of the length of the actuator, it is contented with the simple proportional relation between the flexure and the charge, and discontinues just when the phenomenon of back-relaxation becomes noticeable. The objective of the current paper is to proceed from that stage of development.

One of the advantages of the approach of RC transmission line is the presence of the analytical solutions of its response to some specific input signals, particularly to the Heaviside step function. This enables easy comparison of the model with the real response of the actuator to the input voltage signal representing a flat step up. Moreover, it is possible to attach additional contacts to the surface of the actuator, and in this way validate the electrical parameters of the transmission line [16, 22]. The propagation of voltage along this RC line is described by the following PDE:

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Fortunately, it is possible to solve this PDE analytically for utilizing the method of separation of the variables [16, 23]. Moreover, by applying the initial and boundary conditions – initially the line is discharged and its other end is open – the analytical expression describing the time-evolution of the voltage distribution on the line after applying a voltage step to its input is

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where L is the length of the sample,

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It turns out that the complicated time-evolution of the voltage distribution on the line is defined solely by the measurable parameters: by impedance of the material, by the length of the sample, and by time. Although the PDE describing the transient transferred charge is analogous to (13), it is impossible to apply the boundary conditions and obtain the solution, similar to (14). Instead, if initially the line is discharged, the charge is expressed via voltage as

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As an illustrative example, the simulated transient behavior of voltage and charge of the actuator depicted in figure 1, are given in figure 8. The electrodes of this actuator are chemically deposited platinum of relatively good conductivity. Due to the notable value of the conductivity , the steady distribution of charge and voltage never reach the uniform level. This leads to the non-uniform bending of this actuator. As seen in figure 1, it bends more actively close to the input contacts, while the region close to its tip remains almost unbent.

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| (a) | (b) |
| **Figure 8**. Voltage (a) and charge (b) profiles for a typical IPMC actuator. | |

*5.2. Distributed viscoelastic electromechanical model of iEAP actuators.*

The illustrative sketch of our viscoelastic electromechanical model of iEAP actuators given in figure 9 consists of infinite series of infinitesimally short lumped units, similar to those described in section 3. The electrical conductivity between the units is assured by the opposite electrodes of the iEAP material. The resulting electrical equivalent circuit of the whole device is the RC transmission line presented in the previous section. The mechanical constituent of the model resembles a serial robotic manipulator with active joints powered according to our model of viscoelasticity. The bending moments between the links are determined by the charge of the capacitance within the particular lumped unit, and by the parameters of the model of viscoelasticity presented in section 2. Hereinafter we demonstrate that the proportional correlation between the electric charge and the input moment of our viscoelastic element matches perfectly with the experimental results carried out with several different iEAP materials.



**Figure 9**. The coupling between the electrical equivalent circuit and mechanical constituent.

In section 2 we have declared that input of our viscoelastic element is the strain . When the length of the short unit approaches zero, the strain can be substituted by the according bending moment, proportional to the charge by the coefficient :

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The modulus of elasticity and the second moment of area of the homogeneous rectangular actuators are invariable. Grouping all coefficients into a single factor denoting the proportionality between the curvature and charge as

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the equation (6) yields the time- and coordinate-dependent radius of curvature as

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where the charge is obtained according to (17).

In the Cartesian coordinates the curvature is expressed as

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where

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and are the angle and position of the curve at respectively.

**6. Verification of the model**

In order to prove the validity of the viscoelastic model, we present the actual behavior of different iEAP actuators, and the computational results of the simulations using (20) and (21). The actuators with the input contacts at one end were subjected to the voltage defined by the Heaviside step function. In order to eliminate the moment produced by the own weight of the actuators, the samples are positioned edgewise, illuminated from underneath, and recorded from above by a CCD camera. The image processing was performed later, according to the technique described hereinabove.

The experiments were carried out with the actuators sliced from three different iEAP materials:

1. The **iEAP-A** is a conventional wet IPMC with nafion membrane and platinum electrodes. It contains water; the cations introduced were Na+. Although it is intended for continuous work in water environment, the measurements of short duration were passed in air. The accordance of its transmission line parameters ensures that the steady voltage distribution is gained within a second. This material is soft, gains its maximal actuating amplitude in less than a second and relaxes back in about 10 seconds. Its working voltage should not exceed its electrochemical window – about 1.7V. The actuator presented in figure 1 is a typical example of this iEAP material.
2. The **iEAP-B** is a so-called carbon-polymer composite (CPC). Its membrane is a non-ionic polymer – PVdF(HFP), while the electrodes are made of nanoporous carbide-derived carbon with PVdF(HFP) binder. The whole laminate contains ionic liquid (EMIBF4) and is fabricated by hot-pressing. Due to the huge capacitance and low conductivity of the nanoporous carbon electrodes it is slow, gaining its ultimate bending amplitude in about 60 seconds and relaxing back up to 600 seconds. Its ultimate bending amplitude is not as high as that of the previous material, but it produces much stronger blocking force than the other two. Its working voltage – up to 2.8V – is defined by the ionic liquid used. For details please refer to [24].
3. The **iEAP-C** is an intermediate between the others. Its membrane is nafion and electrolyte is an ionic liquid EMITf, but the electrodes are made of the nanoporous carbide-derived carbon and covered with 15 µm of gold. So, by means of the membrane it is IPMC, but by means of the electrodes it is CPC. The uppermost gold layer guarantees the good conductivity of the electrodes that in turn determines the speed and strength of this material. It produces considerable amplitude of bending in the hertz-range frequency, gains its maximum amplitude in a few seconds and relaxes back in less than a minute. The detailed description of fabrication and properties of this iEAP material is available in [25] referred to as the Carbon(1).

*6.1. Determination of the parameters*

The first stage in the process of the verification of the model is the determination of the parameters. Naturally, the set of parameters of a sample are determined only once before the experiments. The parameters are of two types – electrical and mechanical. The electrical parameters include the resistances of the electrodes and , and the impedance of the material characterized by the capacitance and two conductivities and . While all electrical parameters are interpreted as per unit of length of the sample, a specific scaling parameter of this model is the length of the sample. The mechanical parameters are the rate of relaxation , and the coefficient between the curvature and charge, introduced with (19).

The resistance of the electrodes can be measured with ohmmeter and 2- or 4-terminal sensing method, while the voltage should be kept within the electrochemical window of the electrolyte. It should be easy to characterize the set of the impedance parameters , and with a potentiostat. Unfortunately the sets of the parameters of the used iEAP materials exceed the range of the ordinary electrochemical equipment. Due to the huge capacitance, the period of the exciting sinusoidal voltage has to be from hour up to a day. As an alternative, these parameters were determined by fitting the step response of the propagation of voltage along the samples.

The whole set of the parameters was determined in 3 phases:

1. The resistances of electrodes and were measured using 4-terminal sensing method.
2. The actuator was excited with step voltage while the propagation of voltage along the actuator was measured by attaching 2-3 pairs of additional terminals onto the surface of the actuator, see [16]. The collected data was fitted against (11) in order to obtain the values of , and assuming that , and are known.
3. In section 4 we demonstrated that for a short period after the step input voltage the difference between the behavior of strain and charge is inconsiderable. This presumption helps to determine the mechanical parameters and by fitting the experimentally recorded bending against (20), taking into account all other parameters determined in the steps above.

The fitting was performed using the differential evolution algorithm [26]. The cost function of the optimization was defined as a simple minimization of the differences between experimental and simulated data. The determined parameters of the three samples are given in the table 1.

Table 1. Parameters of the samples.

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| iEAP | **A** | **B** | **C** |
| Dimensions | 45x3x0.24 | 14x1x0.27 | 28x5x0.22 |
| R | 15 | 2510 | 5 |
| C | 5.9 | 10.0 | 65.7 |
| G | 0.007 | 0.00013 | 0.022 |
| W | 0.1 | 0.013 | 0.11 |
|  | 29.5 | 27.7 | 2.1 |
|  | 0.29 | 0.006 | 0.0029 |

*6.2. Validation of the model*

Figure 10 presents the comparisons of the experimentally obtained and simulated curvatures of the three different actuators. The experiments recorded to video are characterized according to the technique described in the section 4.2, while the simulated curvatures are obtained according to (20). The comparison is given in three arbitrary distances from the input contacts only, as adding more graphs makes the figures unreadable. The relative error did not exceed 10% over all experiments. The main causes of the imprecision are the imperfection of the determined curvature and the non-uniformity of the samples, originated from their manufacturing techniques.

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| **iEAP-A** | **iEAP-B** | **iEAP-C** |

**Figure 10**. Comparison of the experimentally obtained and simulated bending of the three materials, each at three distances from the input contacts.

The equation (21) allows determining of the overall shape of the actuators in the course of bending and relaxing back. It affords calculating precisely even the commonly used characteristic of the iEAP actuators – tip displacement – taking into consideration the displacement along both axes. The illustrative example given in figure 11 depicts the actual and simulated shapes of the actuator at five consecutive moments in time: the initial state, bending until the ultimate amplitude at , and two moments in time while relaxing back. The recorded shapes of the actuator are imaged with the solid and dashed lines, while the corresponding simulations are signified with the markers. The iEAP material depicted in this figure is the iEAP-B.



**Figure 11**. Simulated (markers) and recorded (lines) shapes of the bending of the sample iEAP-B.

**7. Discussion**

The actuation of the iEAP actuators is caused by the expansion and contraction of the separate viscoelastic layers of the multilayer material. Figuratively, the displacement is initiated from “inside” of the layers of the laminate, between the viscous and elastic threads of the polymer molecules network. The classical schemes of viscoelasticity – the Maxwell model, Kelvin-Voigt model, and the various combinations of the two – do not describe adequately this situation. Due to the constraint of the application of force, they are unable to describe simultaneously both – the actuation and relaxation – of the iEAP materials. However, a non-traditional approach to the same well-known elements of the traditional viscoelastic schemes – spring and damper – results with a qualitatively new model of viscoelasticity, fulfilling this gap. Combining a series of short entities containing the lumped electrical circuit and the bending moment generated according to the novel viscoelastic scheme results with a distributed model of the iEAP actuators. This assembly resembles a serial robotic manipulator with active joints powered according to our model of viscoelasticity. It takes into account the superposition principle and describes the back-relaxation of the actuators after their forced bending forward. The experiments show that this model is in perfect accordance with the experiments carried out with three iEAP actuators of different electrode and membrane materials.

In order to control an iEAP device properly, not only the history of the electric input signal must be taken into account, but also the history of loading. The elements playing the role of “memory” by electrical and mechanical means are the capacitance and damper correspondingly. Knowing the two histories enables applying the superposition principle – each loading step makes independent contribution to the total loading history by electrical as well as the mechanical means. This principle helps to simulate the distribution of the charges of the capacitances and mechanical moments at any given time. Exploiting the theory of the serial robotic manipulators enables predicting the shape or the force applied by the actuators in more complex configurations than just a simple free bending.

The physical mechanisms of actuation of the above-mentioned iEAP materials are different. The general understanding about the principles of the aqueous IPMC explains the bending effect with the volume change caused by electrolyte [1, 2], while the explanation of the similar effect of the CPC is still under dispute [27]. Even without considering the exact mechanism of expansion, we can argue that the same viscoelastic model describes perfectly the bending-relaxing actuation of both. This analogy leads to a conclusion that the back-relaxation can be explained with the inert distension of the polymer network in the case of all mentioned iEAP materials.

The purpose of the current paper is to introduce the non-traditional approach of the viscoelastic model with the internal strain. Even though the model shows a good accordance with the above-mentioned iEAP materials, there are still several issues open:

* Though the paper [16] describes with a transmission line an aqueous IPMC only, we have seamlessly expanded its scope to the CPC. While the ionic liquid in the membrane can be explained by the impedance consisting of capacitance in parallel with a low conductivity, the only concern is the existence of the capacitive element in the equivalent circuit of the carbonaceous electrodes of CPC. The arguments are given by Kruusamäe *et al.* [28], affirming that there are no capacitive components in the electrodes of the equivalent circuit of CPC.
* Punning *et al.* have demonstrated that the electrical conductivity of the platinum electrodes of IPMC are strongly correlated to the spatial curvature of the material [22]. Kruusamäe *et al*. have found the similar effect upon the carbonaceous electrodes of CPC [28]. The model described in the current paper is simplified to some extent and does not include the variation of the resistance of the electrodes.
* Some authors have noticed that the actuators made of some particular iEAP material relax back only a little bit, but certainly not until the initial state [29, 30]. In the model proposed in the current paper the deformation is modeled in accordance to Hooke’s law only. It can be significantly improved by adding extra nonlinear elements of friction, creep and permanent deformation occurring beyond the elastic limit [31]. By choosing the appropriate rate of relaxation , the model describes perfectly even the actuators for which the back-relaxation is of insignificant value.
* In the scope of the current paper, the mass of the actuator is neglected and each time step is treated as an independent static problem. Adding the elements of inertia, gravity, viscosity of the environment, etc. to the model of viscoelasticity, will certainly enhance the result in certain conditions.
* Using a single effective elastic modulus for entire actuator is appropriate only in the case of symmetric cross-section. In fact, our model is capable depicting the bending of the actuators of arbitrary shape.

These are only a few possible improvements to the model. Supposedly including the whole set of the possible additional parameters analytically with PDE-s is a complicated task. Nevertheless, since the model is fully described with PDE-s and with the measurable physical parameters of the iEAP material, it is rational to implement the whole model with finite elements. While we have an analytical solution for the Heaviside step function only, the finite elements method enables simulating more complicated situations, e.g. the process depicted in figures 1 and 6.

The scope of the proposed model of viscoelasticity with internal strain is not limited with the iEAPs only. Supposedly it should be valid in any context where the displacement is initiated from “inside”, e.g. growing of plants or wetting of plywood. Certainly it could describe the charge-induced dimensional changes of the electric double-layer capacitors. In the sense of assembly, as well as the working principle, these devices are similar to the CPCs. Their difference is based on the optimal congruence of the surface area of pores and the transport of the electrolyte [32]. In actuating devices, the strain and force characteristics are determined by a proper combination of ionic conductivity and elasticity of the composite, while the changes in the dimensions of electrodes of the supercapacitors are undesirable [33].

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