P740.HW3.tex

Due 02/21/07

1. Mass of the atmosphere. As an example of the equation of hydrostatic equilibrium we got the result

$$P(z) = P(0)\ exp - z/z_T, \tag{1}$$

where z_T was the scale height of an atmosphere, $z_T = k_B T/(mg)$. Use this result and the ideal gas law for the case of the earth, e.g., assume $P(0)$ is one atmosphere, $T = 300$ K, and calculate M_a, the total mass of the earths atmosphere.

   (a) Compare M_a to the mass of the earth.

   (b) Compare M_a to the amount of carbon emitted into the atmosphere annually. You'll have to get on the web or ... .

2. Consider a self gravitating body of mass M and radius R for which the equation of hydrostatic equilibrium is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\rho}\frac{\partial P}{\partial r}\right) = -4\pi G\rho. \tag{2}$$

Assume that the material from which this body is constructed obeys the equation of state

$$P(\rho) = \frac{P_0}{2}\left(\frac{\rho}{\rho_0}\right)^2 = \frac{C}{2}\rho^2, \tag{3}$$

where P_0 and ρ_0 are known constants.

   (a) Find the differential equation for ρ. In this equation scale r by R, i.e., use $x = r/R$, and identify the dimensionless variable that controls the solution. Note that the equation is *linear* and *homogeneous* so the amplitude of the solution is not fixed.

   (b) Solve for $\rho(r)$. You should notice that part of the ODE you have looks much like Schroedinger for the hydrogen atom. Your quantum text might help here. Find ρ and fix the amplitude by requiring

$$M = \int_0^R 4\pi r^2 \rho(r)dr = 4\pi R^3 \int_0^1 x^2\rho(x)dx, \tag{4}$$

   or

$$\overline{\rho} = \frac{M}{\frac{4}{3}\pi R^3} = 3\int_0^1 x^2\rho(x)dx. \tag{5}$$

(c) Make a plot of $\rho/\overline{\rho}$ as a function of x.

3. This is a jazzed up version of the calculation in Section I.B of **P740.1.tex**. Start with the full set of fluid dynamics equations, Eqs. (33)-(35) of **Afternote.2.tex**. Consider the case where in equilibrium the system is at $(\rho_0, P_0, T_0, \mathbf{u} = \mathbf{F} = 0)$, $T \propto \theta$.

   (a) Linearize these equations using $(\rho_0 + \delta\rho, P_0 + \delta P, T_0 + \delta T, \delta\mathbf{u} \neq 0, \mathbf{F} = 0)$. You will need Eq. (36) to close these equations.

   (b) Solve these equations for a disturbance with space-time structure

   $$\delta X = A_X \; e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}. \tag{6}$$

   You should find a homogeneous set of linear equations whose solution requires a special relation between $\mathbf{k}$ and ω, the dispersion relation.

   (c) There are two mechanisms for attenuation of wave propagation, one related to viscosity and another related to the thermal conductivity. Are these mechanisms of attenuation additive? Compare them for air at STP.