A mechanical model of a non-uniform Ionomeric Polymer Metal Composite (IPMC) actuator

Mart Anton, Alvo Aabloo, Andres Punning and Maarja Kruusmaa

IMS Lab, Institute of Technology, Tartu University, Nooruse 1, Tartu 50411, Estonia

{Mart.Anton, Alvo.Aabloo, Andres.Punning, Maarja.Kruusmaa }@ut.ee

Abstract
This paper describes a mechanical model of an IPMC (Ionomeric Polymer Metal Composite) actuator in a cantilever beam configuration. Apart from mechanical models developed so far, our model permits modeling the continuously bending surface of the IPMC actuator with a varying bending curvature. This model takes into account large deflections and that the electrically induced bending moment does not have to be constant. We also investigate the case where part of an IPMC actuator is replaced with a rigid elongation and demonstrate that this configuration would make the actuator to behave more linearly.

Subject classification numbers
81.05.Lg (Polymers and plastics; rubber; synthetic and natural fibers; organometallic and organic materials)
82.35.-x (Polymers: properties; reactions; polymerization)
82.35.Lr (Physical properties of polymers)

1 Introduction

IPMC (Ionomeric polymer metal composite) is a type of an electroactive material that bends in electric field [1, 2]. It consists of a thin swollen polymer film, such as Nafion™, filled with water or ionic liquid. Both sides of the polymer film are plated with thin metal electrodes. Voltage applied between the surface electrodes causes migration of ions inside the structure of the polymer, which in turn causes the mechanical bending of the sheet as shown in figure 1. The direction of bending depends on the polarity of the applied electric field.

(a) (b) (c)

Figure 1 IPMC with (a and c) and without (b) electrical stimulation.

This paper investigates the mechanical model of an IPMC sheet in the cantilever beam configuration. Various mechanical and electromechanical models have been developed by a number of authors, emphasizing different aspects.

In most of cases, the mechanical properties of the sheet are investigated in terms of tip displacement and tip force near the initial position of the tip. For example, in [3, 4, 5, 6] the behaviour of the IPMC beam is described at small displacements. Large deflections are modeled in [7] whereas the curvature of the bending sheet is considered to be constant. In [8] a segmented IPMC actuator is analysed, whereas each segment is independently controlled and the bending curvature of a segment is constant. Continuously changing curvature and large deflection is considered in [9]. However in that model force is applied only in one position, sheet is assumed to be ideally straight and free bending curvature is assumed to be constant.

The mechanical model represented in this paper is different from the studies cited above in that we assume that the bending of the sheet can be non-uniform. We investigate the cases where the deflections are large while the bending curvature does not have to be constant. Thus we do not model solely the tip deflection or the tip force or a curvature in a predefined point but the entire configuration of a continuously bending non-uniform surface. Moreover, we also consider a case, where the IPMC actuator is elongated with a rigid segment.

In addition, our model enables to predict free bending curvature and curvature when an external force is applied to the sheet. Also a case when the actuator is pushing an object is modeled. It permits to find the force applied to the object and curvature of an IPMC sheet. Different from many other studies the object does not have to be near the initial position, but can be in an arbitrary position.

One of the contributions of this work is that we also investigate the IPMC actuator with the rigid elongation and the effect of the elongation to the performance and properties of the actuator. Previously the use of elongation has been studied in [10], but model developed there is only accurate in case of small deflections. Our model permits describing an IPMC sheet with a rigid elongation and compare it to the conventional IPMC actuator. The comparison is done both experimentally and theoretically with the help of the model. Experimental data and simulation results are consistent. It turns out that the long IPMC sheet and the short IPMC sheet with elongation are under many circumstances capable of equal performance. Moreover, the elongation makes the actuator behave linearly and variation of parameters along the IPMC sheet can be neglected.

2 The model

This section represents the mechanical model of the IPMC cantilever actuator with the properties given above.

Our objective is to model a situation, where an IPMC sheet in a cantilever configuration applies force to an object (load cell, for example). The sheet can have an absolutely rigid elongation attached to the tip. In this study we model the sheet in a static equilibrium state (the sheet does not move).

The shape of an IPMC sheet is modeled as a curve in 2D space (see figure 2). The curve is a projection of the neutral surface – the surface that does neither contract nor expand. Throughout this paper s denotes the natural parameter of the curve, what specifies a position on the sheet at a distance s from contacts along the curve. Neutral curve can be uniformly defined with its curvature $k(s)$ (see figure 3). For definition details please refer to appendix.

A part of the IPMC sheet is fixed between contacts at clamps or the elongation. This part does not influence the sheet behaviour. The freely bending IPMC part of the sheet is characterized by the following parameters

1. length - l,
2. initial curvature - $k_0(s)$,
3. beam stiffness - D,
4. electrically induced bending moment (shortly EIBM) - $M_e(s)$.

Parameters $k_0(s)$ and $M_e(s)$ are only defined for positions $s \in [0, l]$. For the elongation they are undefined. If we assume, that $M_e(s)$ does not vary along the sheet, the notation M_e may be used instead. In case no force is applied and there is no electrical stimulation $M_e = 0$ and $k_0(s) = k(s)$.

The same model can be used for modeling both the actuators with and without elongation. Formally every IPMC sheet is considered to have an elongation (see figure 2). It helps to model elongated sheets but can equally be used for modeling sheets without an elongation. Elongation starts from a position l. It is absolutely rigid and straight so curvature $k(s)$ for the elongation of the sheet is always zero:

$$\forall s > l \quad k(s) = 0 \tag{1}$$

The object is located on a circular trajectory in a position p (see figure 3) where p specifies a position on trajectory at a distance p from zero position along the trajectory. The center of the circle is at the contacts of the sheet and the radius is denoted as R. The sheet is assumed to be in a frictionless contact with the object thus reaction is perpendicular to the sheet. Force applied to the sheet is denoted with F_{sheet}. According to Newton's third law the force applied to the object is $-F_{\text{sheet}}$. The force applied to the object in direction of the tangent of the trajectory is

$$F = -F_{\text{sheet}} \cdot \cos(\phi) \tag{2}$$

, where ϕ is the angle between the normal of the sheet and the tangent of the trajectory at the position of the object. For the rest of the paper if not specified otherwise by force we mean F.

In this paper we study the relations between

1. IPMC sheet properties - l, $k_0(s)$, D, $M_e(s)$;
2. curvature of the sheet - $k(s)$;
3. radius of the trajectory - R;
4. position on the trajectory - p;
5. force applied to the object - F.

The exact set of equations defining the relationship between the listed notations is given in appendix.

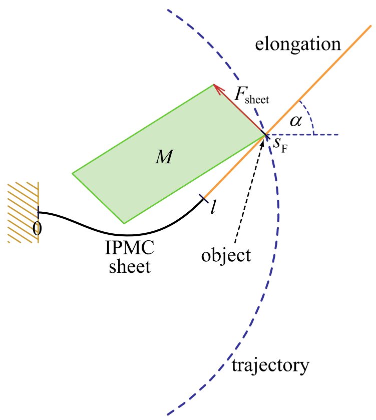


Figure 2 Actuator – notations for simplified case.

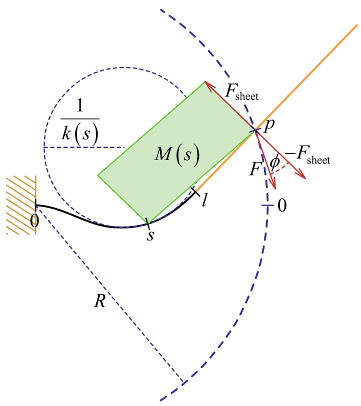


Figure 3 Actuator – notations for general case.

Parameters $k_0(s)$, D and $M_e(s)$ relate to curvature $k(s)$ as follows:

$$\forall s \le l: \quad k(s) - k_0(s) = \frac{M_e(s) + M(s)}{D} \tag{3}$$

, where $M(s)$ is the bending moment caused by the force F. $M(s)$ is equal to the area of the parallelogram in figure 3. Please refer to appendix for the mathematical definition of $M(s)$.

If we assume that EIBM is constant ($M_e(s) = M_e$) it holds that:

$$\frac{\alpha - \alpha_0}{\min(l, s_F)} = \frac{M_e + M}{D} \tag{4}$$

, where notations have the following meanings (see figure 2):

- M is the mean bending moment caused by the force F in the loaded part of the bendable section (equal with the area of the parallelogram on figure 2),
- s_F is the position where the force F is applied,
- α is the angular deflection at that position,
- α_0 is the initial angular deflection at the same position.

Please refer to appendix for mathematical definitions of these notations and derivation of (4).
Note that if $F = 0$ and $M_e = 0$ then $\alpha = \alpha_0$.
If we double the width of the IPMC sheet, it is equivalent to two sheets working in parallel. Force needed to bend would double. The same holds for stiffness and EIBM. Output force F, beam stiffness D and EIBM $M_e(s)$ are proportional with the width w of the sheet. D and $M_e(s)$ normalized to the width of the sheet characterize the properties of the IPMC material.

3 System setup

This section describes the experimental setup to

1) investigate properties of the IPMC material,
2) compare the energy and energy per area of IPMC sheet of actuators with and without elongation,
3) verify the model introduced in the previous section.

Our system consists of six basic parts (see figure 4):

1. A sheet either consisting entirely of IPMC material or a combination of IPMC material and a plastic elongation attached to it with bolt and nut.
2. A clamp which holds the sheet. The jaws of the clamp are made of gold and serve as an electrical contact. It also holds the sheet so that it bends in the horizontal plane. The horizontal motion assures that gravity does not affect the experimental results.

3. A Transducer Techniques GSO-10 load cell is used to measure the force applied by the sheet. Maximum force that can be measured is about 0.1N. The sheet is held between two rolls, to achieve a frictionless contact.
4. Black&white camera Dragonfly EXPRESS is used to record the sheet. The resolution of the camera is 640×480 pixels and frame rate used is 25 fps. From acquired images the neutral curve can be determined.
5. The inputs and outputs of the system are controlled and monitored with a PC running LabView 7. The input voltage of the sheet is controlled with National Instruments I/O board PCI-6703. Load cell is attached to DAQ board PCI-6034. The camera Dragonfly EXPRESS is connected to the IEEE 1394b port.
6. The control signal generated by I/O board is amplified with the amplifier LM675 (National Semiconductor).

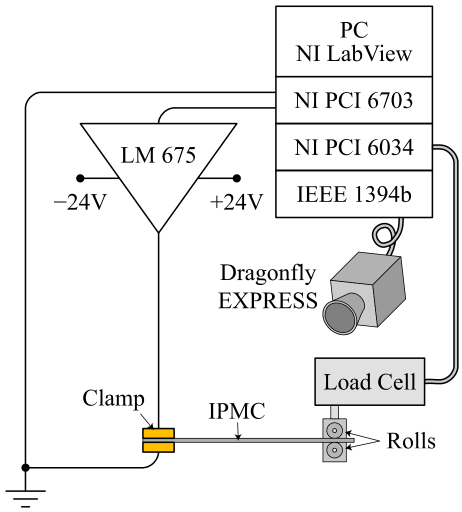


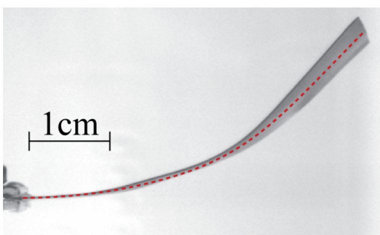
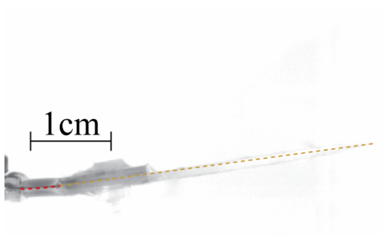
Figure 4 System setup.

4 Experiments

4.1 Design of the experiments

System setup presented in previous section allows us to use an arbitrary sheet, apply voltage to it and measure force and neutral curve in arbitrary position of the object (in our case the load cell).
Table 1 summarises information about the two sheet configurations used in experiments. Long IPMC sheet is initially twisted but will be straight when held between rolls. The short IPMC sheet is cut from the first part of the long IPMC sheet after conducting the experiments with the long one.

Table 1 Sheet configurations.

Configuration	Long IPMC sheet	Short IPMC sheet with the plastic elongation
Side view of the sheet and the initial neutral curve (the red line) with curvature - $k_0(s)$.		
Length of the freely bending IPMC part - l	50 mm	4.5 mm
Length of the part of the IPMC sheet that is fixed - l_c	1.5 mm	3.5 mm

Width of the IPMC sheet - w	11 mm	11 mm
Thickness of the IPMC sheet - d	0.21 mm	0.21 mm

Parameter l_c specifies the length of the IPMC sheet that is fixed between contacts at clamps or elongation. This parameter does not influence the behavior of the sheet. Parameters w and d, on the other hand, have a direct impact to the sheets behavior. They are not explicitly used in the model but model parameters D and $M_e(s)$ are related to them. They are measured to assess the parameters of the IPMC material.

Throughout this paper we call actuators corresponding to sheets introduced "long sheet" and "short sheet" respectively. Throughout this paper the radius of the trajectory is $R = 40\,\text{mm}$. In experiments force is measured in 7 positions located on the trajectory (see figure 5).

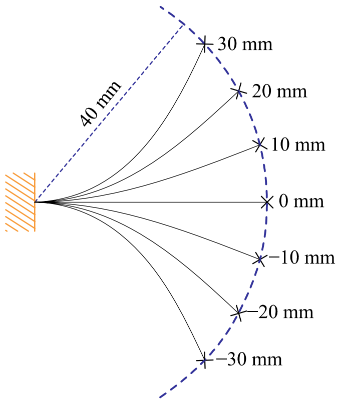


Figure 5 Positions on trajectory, where force and shape of the sheet are measured. Measure denotes distance along the trajectory form position zero.

For each of the:
- two sheet configurations,
- 7 trajectory positions and
- 3 driving voltages (+2V, 0V and −2V)

force F and the neutral curve of the sheet are measured. From the neutral curve, the curvature $k(s)$ can be extracted.

4.2 Implementation details

For every sheet configuration and position of the trajectory the force and the neutral curve at 3 driving voltages (+2V, 0V and −2V) is measured. Voltage input and timing of important readings can be seen in figure 6. First 10 readings are taken at 0V. After these 10 readings, the readings are taken alternatively at +2V and -2V. Force and neutral curve are measured 0.5s after the start of the impulse. To enable signal filtering the total length of the input pulse is 0.6s.

Electric current at 2V dehydrates the IPMC sheet [1, 11]. Changes in the hydration level of the IPMC sheet cause changes in beam stiffness and response to electrical stimulation. Therefore the experiments where kept as short as possible and the IPMC sheet was moistened between the measurements.

After every pulse water is unevenly distributed within the material. The first 2 pulses are neglected because initial conditions are different compared to those of the following pulses.

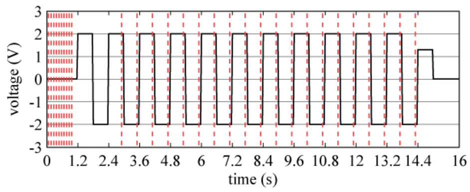


Figure 6 Input voltage and times of important readings.

We measure force component in direction of the tangent of the trajectory. The load cell measures the component of the force that is perpendicular to it and has to be positioned perpendicular to the tangent of the trajectory. Ten readings are taken and the mean value is used as the result.

Ten images are saved synchronously with force measurements. Mean of these images as light intensity matrixes is computed. This operation may cause motion blur. Neutral curve of the IPMC part of the sheet is interpolated with minimum variation curvature curve [12]. Curvature of such a curve can be easily analytically revealed.

4.3 Experimental data

Figure 7 represents the neutral curves measured. Measured forces are presented on figure 9. Videos of the experiments in position 0 (of the trajectory) corresponding to long and short sheet can be downloaded from https://www.ims.ut.ee/mediawiki/upload/1/13/IPMC_Mechanics_Long.avi and https://www.ims.ut.ee/mediawiki/upload/6/63/IPMC_Mechanics_Short.avi respectively.

On the videos and in figure 7 can be seen, that with different voltages the static equilibrium state of the sheet differs. In case of a long sheet the difference is big. In case of a short sheet with elongation the difference is barley notable. When considering moving sheets the shape of the sheet is an input parameter that we need to know in order to calculate the output force. In case of short sheet with elongation the shape of the sheet does not vary so much and can be easily estimated. In case of the long sheet estimation of the shape of the sheet is more difficult.

Readings taken in position 30 mm of the trajectory are invalid because the sheet systematically got stuck between the rolls.

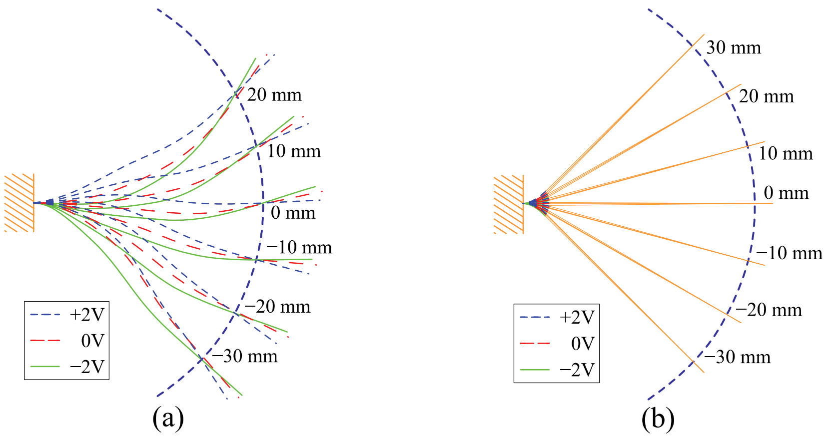


Figure 7 Neutral curves of long sheet (a) and short sheet (b).

5 Parameter extraction, model validation and simulations

In this section we validate the model of the IPMC actuator by extracting the parameters for the model and comparing the simulation result with the experimental results of the previous section.

5.1 Parameter extraction

The parameters that have to be extracted from the experiments for both of the sheet configurations are beam stiffness and the electrically induced bending moment (EIBM).Each of three driving voltage, used in the previous section to drive the actuator, cause a different EIBM but the beam stiffness is always the same.

At 0V $M_e(s)=0$ (no EIBM). The following equation derived from (4) can be used to calculate beam stiffness:

$$D=\frac{\min(l,s_F)\cdot M}{\alpha-\alpha_0} \quad (5)$$

There are number of measurements (corresponding to different positions on trajectory) that each give sample of beam stiffness. The weighted average is used as the result. A weight coefficient M is used so that the experiments with a higher mean bending moment would contribute more to the result.

Table 2 summarizes the notations for EIBM-s at two electrical stimulations (0V excluded). We are allowing EIBM to vary along the sheet and also consider an assumption that it is constant.

Table 2 Notations for EIBM.

Electrical stimulation	Possibly varying EIBM	Constant EIBM
+2V	$M_e^+(s)$	M_e^+
−2V	$M_e^-(s)$	M_e^-

Varying electrically induced bending moment in case of both electrical stimulations can be calculated using equation derived form (3):

$$M_e(s)=D\cdot(k(s)-k_0(s))-M(s) \quad (6)$$

For calculating the constant electrically induced bending moment the equation derived form (4) can be used:

$$M_e=\frac{D\cdot(\alpha-\alpha_0)}{\min(l,s_F)}-M \quad (7)$$

In both cases (varying and constant EIBM) there are again number of measurements that each give sample of EIBM. Arithmetical average is used as the result.

5.2 Extracted parameters

Extracted parameters are presented in table 3 and in figure 8.

IPMC is a sandwich with cracked surface and several layers with different elastic modulus. The Young modulus of a homogenous sheet with same dimensions and similar stiffness can be calculated using the equation

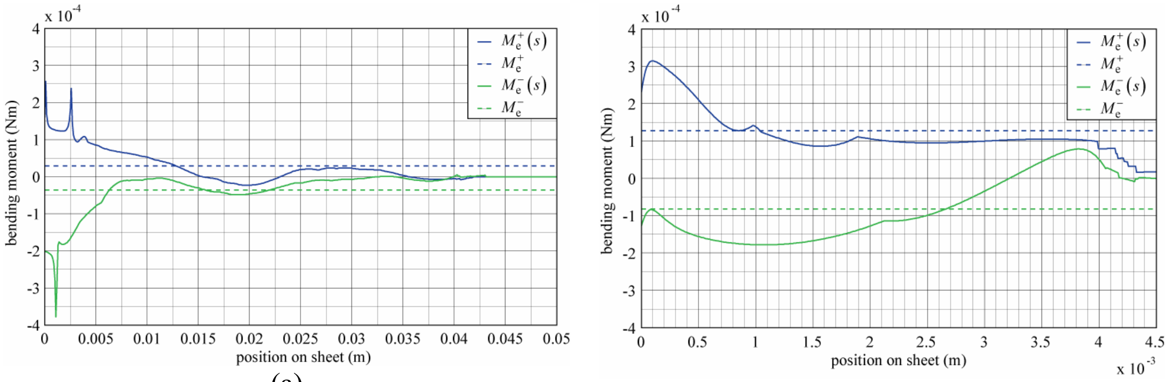
$$E=\frac{12\cdot D}{w\cdot d^3} \quad (8)$$

This parameter is also called effective Young modulus or equivalent Young modulus. It's a classical beam characteristic. Equivalent Young modules for our sheets can be found in table 3.

Table 3 Extracted parameters.

Configuration	Notation (equation)	Long IPMC sheet	Short IPMC sheet
Beam stiffness	D	$2.03\cdot10^{-6}$ N·m^2	$1.21\cdot10^{-6}$ N·m^2
Beam stiffness normalized to the width of the sheet	$\frac{D}{w}$	$1.84\cdot10^{-4}$ N·m	$1.10\cdot10^{-4}$ N·m

Equivalent Young modulus	E	236 MPa	147 MPa
Constant EIBM at +2V	M_e^+	0.029 mN·m	0.127 mN·m
Constant EIBM at −2V	M_e^-	−0.036 mN·m	−0.082 mN·m
The mean absolute value of constant EIBM	$\frac{M_e^+ - M_e^-}{2}$	0.032 mN·m	0.104 mN·m
The mean absolute value of constant EIBM normalized to the width of the sheet	$\frac{M_e^+ - M_e^-}{2 \cdot w}$	2.95 mN	9.47 mN



(a) (b)

Figure 8 Electrically induced bending moment of long sheet (a) and short sheet (b).

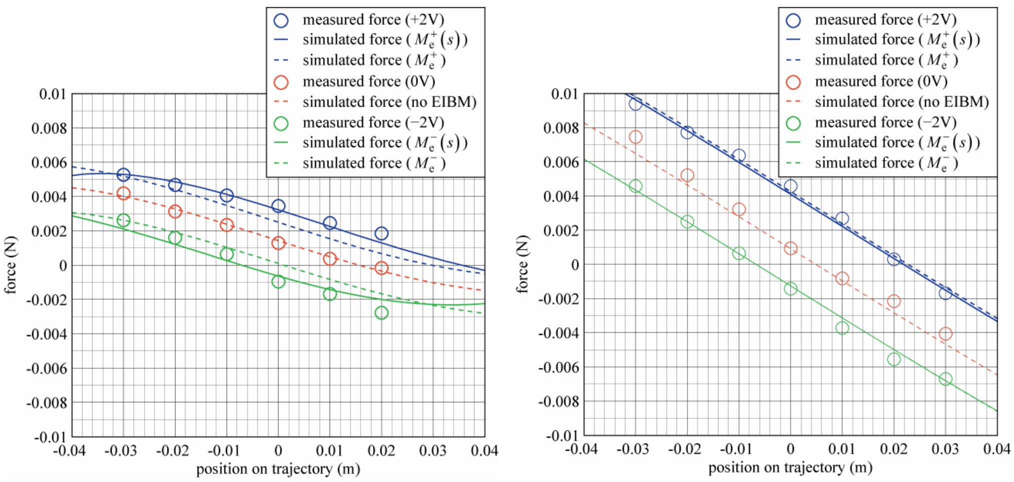
Observations

1) Long sheet appears to be stiffer then short sheet (although the short IPMC sheet is cut from the first part of the long IPMC sheet). It can be explained by changes in hydration level [1, 13]. Short IPMC sheet is surrounded by water while long sheet is not (for side view of the sheet see table 1). Short IPMC sheet is surrounded by water because elongation is near to the contacts and because surface tension of water.
2) As s increases the varying EIBM tend to converge to zero. It can be explained by the fact that moving toward the top of the sheet potential between surfaces drop because of the surface resistance [14]. The smaller potential, the smaller EIBM.
3) The mean absolute value of a constant EIBM is smaller for the long sheet. The top part of the long sheet has small EIBM and thereby the makes sheet weaker as the whole.
4) EIBM (both varying and constant) at voltages +2V and –2V do not have the same absolute value. This reflects the asymmetry of the material.
5) Varying EIBM changes polarity or has peaks at some positions. This is caused by errors in curvature measurements.

5.3 Model validation

With the parameters extracted in the previous section and the parameters from the section 4.1 the simulations are conducted and the simulation results are verified to the experimental results. The algorithms used in this simulation are described at https://www.ims.ut.ee/mediawiki/upload/6/69/IPMC_Mechanics_Algorithms.pdf.

In figure 9 real and simulated output forces at different electrical stimulations can be seen. In table 4 real and simulated neutral curves at +2V in position 0mm of the trajectory can be seen. Root mean squared error of the neutral curve for the IPMC part of the sheet is also given. Table 5 gives root mean squared errors of the neutral curve and of the output force over all positions of trajectory.



(a) (b)

Figure 9 Real and simulated forces of long sheet (a) and short sheet (b).

Table 4 Real and simulated neutral curves and root mean squared errors at +2V in position 0m of the trajectory.

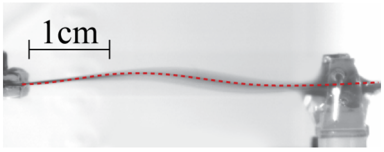
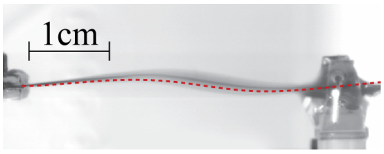
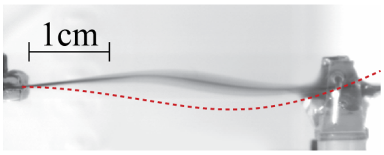
Configuration	Long IPMC sheet	Short IPMC sheet with plastic elongation
Actual	 RMSE = 0 mm	 RMSE = 0 mm
With varying EIBM.	 RMSE = 0.44 mm	 RMSE = 0.05 mm
With constant EIBM.	 RMSE = 2.23 mm	 RMSE = 0.09 mm

Table 5 Root mean squared errors.

Configuration		Long IPMC sheet	Short IPMC sheet with plastic elongation
Varying EIBM	Neutral curve	0.81 mm	0.13 mm
	Force	0.322 mN	0.424 mN
Constant EIBM	Neutral curve	2.13 mm	0.18 mm
	Force	0.622 mN	0.422 mN

Varying EIBM-s ($M_e^+(s)$ and $M_e^-(s)$) are calculated with (6) based on curvature. Constant EIBM-s (M_e^+ and M_e^-) are calculated with (7) based on deflection angle. It is easier to measure the angle than the curvature but comparison of the experiments and simulations indicates that in case of the long sheet the use of constant EIBM causes larger modeling errors (see table 5). In case of the short sheet, the use of constant EIBM gives as good results as the use of varying EIBM does.

Position-force relation is linear in case of the short sheet. This is important with respect to precision control of such an actuator. As controlling linear time invariant system (LTI) is very easy, stabilizing nonlinear system (like actuator with long sheet) can be very difficult.

The initial position of the object (without electrical stimulation) is 15mm in case of the long sheet and 5mm in case of the short sheet (see figure 9). The short sheet can apply more force in the initial position, than long sheet is able to. The main reasons are:

a) most of the of the work is done by IPMC material near the input contacts, where the bending moment caused by the external force is the largest and
b) EIBM tends to converge to zero in direction away from the contacts.

6 Possible applications and future work

The model presented and validated in this paper can be used to determine the properties of the actuator and to optimize the construction of the actuator.

To determine the properties of the actuator we need to know l, $k_0(s)$, D and R. For every position p of the trajectory and for every EIBM $M_e(s)$ the model determines the shape of the sheet $k(s)$ and the force F applied to the object. Please refer to https://www.ims.ut.ee/mediawiki/upload/6/69/IPMC_Mechanics_Algorithms.pdf for the corresponding algorithm.

Usually we are only interested in a specific section of positions of the trajectory, the working section and a set of EIBM-s determined by the material. Given the working section and a set of realistic EIBM-s the model enables us to do the following:

1. To find the linear fit of the force F as a function of p in the given working section. If the mean squared error is small enough the linear fit can be used for creating a linear controller.
2. To find the maximal difference of the shapes of the sheet at extreme opposite realistic EIBM-s. The less the shape of the sheet varies the easier it is to control it.
3. To find the minimal length of the sheet, capable to reach the object in every position of the working section.
4. To find the maximal force that can be applied in the direction of the tangent and the normal of the trajectory.
5. To find the maximal magnitude of the force that can be applied in both directions within the working section.

Further on, the model also enables us to find the minimal area of an IPMC sheet, capable of applying a predefined force within a predefined working section. As the force per area of the sheet does not depend on the width of the sheet, the required parameter is the optimal length of IPMC sheet. After the optimal length is found, the width of the sheet can be chosen according to the required output force. To investigate how and when the elongation as a construction element improves the performance of the actuator and the optimal configuration of the actuator is the topic of our active research.

7 Conclusions

We presented a model of a cantilever IPMC actuator with a varying electrically induced bending moment. We validate the model against two actuators: a long IPMC sheet and a short IPMC sheet with a rigid elongation. The experimental results show that for the short sheet the position-force relationship is linear whereas the position-force relationship of the long sheet is non-linear.
This paper presents a static analysis of IPMC mechanics. It enables to compute free bending curvature and force applied to an object in a static equilibrium state. A dynamic model which considers motion, masses and viscoelasticity, is a subject of future study. However, for cases where accelerations and

velocities are small enough, the static model is shown do be accurate. It is also suitable for finding the optimal design of the actuator.

Acknowledgements
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Appendix
In section 2 the mechanical model of the IPMC cantilever actuator is introduced. In this appendix a list of parameters is given. Also the meaning and relation of these parameters is clarified.

Appendix.1. Operators

We need following operators to formulate relations between our parameters.

Tangent vector:

$$\vec{\mathrm{T}}(\theta)=\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \tag{9}$$

Normal vector:

$$\vec{\mathrm{N}}(\theta)=\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} \tag{10}$$

Let $A_1=\begin{pmatrix}x_1\\ y_1\end{pmatrix}$, $A_2=\begin{pmatrix}x_2\\ y_2\end{pmatrix}$ be vectors and θ an angle between them.
Definition for cross product in 2D space:

$$A_1\times A_2=|A_1|\cdot|A_2|\cdot\sin(\theta)=x_1\cdot y_2-y_1\cdot x_2 \tag{11}$$

Appendix.2. Parameters

Table 6 Parameters.

Notation	Figure	Description
l	3	Length of freely bendable IPMC section
s_F	2	Position on the sheet, where force is applied
$\hat{s}$	-	Length of loaded (free, bendable) IPMC section
D	-	Beam stiffness
F_{sheet}	3	Force applied to the sheet
F	3	Force applied to the object
ϕ	3	Angle between the normal of the sheet and the tangent of the trajectory
R	3	Radius of the trajectory
p	3	Position of the object on the trajectory
$k_0(s)$	-	Initial curvature function
$\alpha_0(s)$	-	Initial tangential angle function
α_0	-	Initial angular deflection
$k(s)$	3	Curvature function
$\alpha(s)$	10	Tangential angle function
α	2	Angular deflection
$\vec{P}(s)$	10	Position vector function

$\vec{P}$	10	Position vector of the center of the loaded section of IPMC
$M(s)$	3	Bending moment caused by force applied to the sheet
M	2	Mean bending moment of the loaded section of IPMC caused by force applied to the sheet
$M_e(s)$	-	Electrically induced bending moment (EIBM)
M_e	-	Constant EIBM

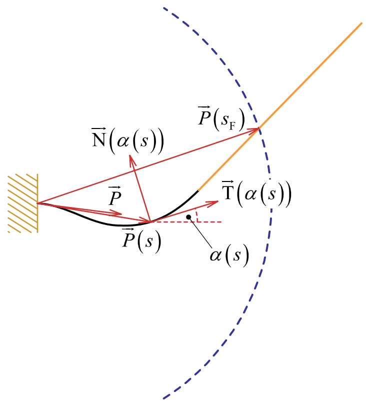

Figure 10 Actuator – complementary notations.

Appendix.3. Relations

The parameters listed in table 6 are related through the following equations:

$$\hat{s} = \min(l, s_F) \tag{12}$$

$$F = -F_{\text{sheet}} \cdot \cos(\phi) \tag{13}$$

$$\phi = \alpha(s_F) - \frac{p}{R} \tag{14}$$

$$\vec{P}(s_F) = R \cdot \vec{T}\left(\frac{p}{R}\right) \tag{15}$$

$$\alpha_0(s) = \int_0^s k_0(u) \cdot du \tag{16}$$

$$\alpha_0 = \alpha_0(s_F) \tag{17}$$

$$k(s) = \begin{cases} k_0(s) + \dfrac{M_e(s) + M(s)}{D} & s \le l \\ 0 & s > l \end{cases} \tag{18}$$

$$\alpha(s) = \int_0^s k(u) \cdot du \tag{19}$$

$$\alpha = \alpha(s_F) \tag{20}$$

$$\vec{P}(s) = \int_0^s \vec{T}(\alpha(u)) \cdot du \tag{21}$$

$$\vec{P} = \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} \vec{P}(u) \cdot du \tag{22}$$

$$M(s) = \begin{cases} \left(\vec{P}(s_F) - \vec{P}(s)\right) \times \left(F_{\text{sheet}} \cdot \vec{N}(\alpha(s_F))\right) & s \le s_F \\ 0 & s > s_F \end{cases} \tag{23}$$

$$M = \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} M(u) \cdot du \tag{24}$$

For realistic results $D > 0$, $R > 0$ and $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$.

Appendix.4. Theorems

Theorem 1

The mean bending moment caused by external force equals to the bending moment in the center of the bendable section.

$$M = \left(\vec{P}(s_\mathrm{F}) - \vec{P}\right) \times \left(F_\mathrm{sheet} \cdot \vec{\mathrm{N}}(\alpha)\right) \tag{25}$$

Proof

$$M = \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} M(u) \cdot du = \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} \left(\vec{P}(s_\mathrm{F}) - \vec{P}(u)\right) \times \left(F_\mathrm{sheet} \cdot \vec{\mathrm{N}}\left(\alpha(s_\mathrm{F})\right)\right) \cdot du =$$

$$= \left(\vec{P}(s_\mathrm{F}) - \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} \vec{P}(u) \cdot du\right) \times \left(F_\mathrm{sheet} \cdot \vec{\mathrm{N}}(\alpha)\right) = \left(\vec{P}(s_\mathrm{F}) - \vec{P}\right) \times \left(F_\mathrm{sheet} \cdot \vec{\mathrm{N}}(\alpha)\right)$$

Theorem 2

If $M_\mathrm{e}(s) = M_\mathrm{e}$, (electrically induced bending moment is constant) then

$$\frac{\alpha - \alpha_0}{\hat{s}} = \frac{M_\mathrm{e} + M}{D} \tag{26}$$

Proof

Assuming $s \le \hat{s}$ and $M_\mathrm{e}(s) = M_\mathrm{e}$ from equation (18) we get

$$k(s) - k_0(s) = \frac{M_\mathrm{e} + M(s)}{D}.$$

Let us integrate

$$\int_0^{\hat{s}} \left(k(u) - k_0(u)\right) \cdot du = \int_0^{\hat{s}} \frac{M_\mathrm{e} + M(s)}{D} \cdot du,$$

$$\int_0^{\hat{s}} k(u) \cdot du - \int_0^{\hat{s}} k_0(u) \cdot du = \frac{\hat{s} \cdot M_\mathrm{e} + \int_0^{\hat{s}} M(u) \cdot du}{D},$$

$$\frac{\int_0^{\hat{s}} k(u) \cdot du - \int_0^{\hat{s}} k_0(u) \cdot du}{\hat{s}} = \frac{M_\mathrm{e} + \frac{1}{\hat{s}} \cdot \int_0^{\hat{s}} M(u) \cdot du}{D},$$

$$\frac{\alpha(\hat{s}) - \alpha_0(\hat{s})}{\hat{s}} = \frac{M_\mathrm{e} + M}{D}.$$

Note that $\alpha(\hat{s}) = \alpha(s_\mathrm{F}) = \alpha$ and $\alpha_0(\hat{s}) = \alpha_0(s_\mathrm{F}) = \alpha_0$.

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