P740.HW7.tex

Due 04/30/07

1. HW99

From the data in the figure **Hwy 99, lane 3**

1. Calculate the speed of the shock, D.

2. Calculate F, the car current (cars/time), to the left of the shock and to the right of the shock.

3. Calculate ρ the density of cars (cars/length) to the left and right of the shock.

4. Use your results for F to predict the speed of the shock from the first Hugoiot relation.

5. Why is $D < 0$?

To carry out the calculations of F and ρ you might divide $d-t$ space into boxes of size 40 m $\times$ 20 sec. F is the rate at which cars enter/leave a box. The density could be calculated by counting the number of data points in a box and scaling properly. Or $\cdots$.

2. dx/dt

Carry out the integration of Eq. (21) in Note 14 (note typo, the equation is valid for $t > t_0 = -x/v_m$).

1. Plot x as a function of t.

2. Consider two cars in the pack behind $x = 0$. If their separation in the pack is L at $t = 0$ how does the distance between them evolve in time? At short time? At long time?

3. Nonlinearity and Fourier analysis.

The assertion is that single frequency Fourier analysis fails for nonlinear problems. See page 2 of Note 14. While the context in Note 14 is wave equations the assertion is about nonlinearity. So for illustrative purposes consider the oscillator equation

$$\ddot{Q} = -\alpha^2 Q + \gamma\alpha^2 Q^2 + \lambda A e^{-i\omega t}, \tag{1}$$

where A is the amplitude of a force that drives the oscillator at frequency ω and λ is a parameter (set to 1 eventually) that allows you to keep track of the consequences of the driving force.

1. Assume that Q can be written as a power series in λ

$$Q = \lambda Q_1 + \lambda^2 Q_2 + \lambda^3 Q_3 + \cdots. \tag{2}$$

   Insert this representation into Eq. (1) and by equating like powers of λ derive an equation for Q_1, for Q_2, for Q_3.

2. Solve the set of equations you have for Q_1, for Q_2.

3. With what frequency does Q_2 move. Do not solve for Q_3. But with what frequency will Q_3 move?

4. Equations of State.

A purported EOS for a material is matched with the experimentally determined state of the material by matching the equation for P on the Hugoniot with the equation for P from the EOS. Formally

$$P(\mu, \epsilon)_{\epsilon=\rho_0 u^2/2} = P_H(\mu, u), \tag{3}$$

where $\mu = \rho/\rho_0 - 1$, $\epsilon = \rho_0 E$ and

$$P_H(\mu, u) = \rho_0 \frac{1+\mu}{\mu} u^2, \tag{4}$$

from the second Hugoniot relation.

1. For the ideal gas ($PV = Nk_BT$ and $E = 3k_BT/2m$) the EOS in terms of the relevant variables is

$$P = \frac{mN}{V}\frac{k_BT}{m} = \frac{2}{3}\frac{\rho}{\rho_0}\rho_0 E = (\gamma - 1)(1+\mu)\epsilon. \tag{5}$$

   For this case the EOS is evaluated at $\epsilon = \epsilon_0 + \rho_0 u^2/2$.

   (a) Solve Eq. (3) for u.

   (b) Find P/ϵ_0 as a function of ρ/ρ_0.

2. For a real gas a form of the van der Waals EOS is

$$\left(P + P_0\frac{\rho^2}{\rho_0^2}\right)\left(V - Na^3\right) = Nk_BT, \quad E = 3k_BT/2m. \tag{6}$$

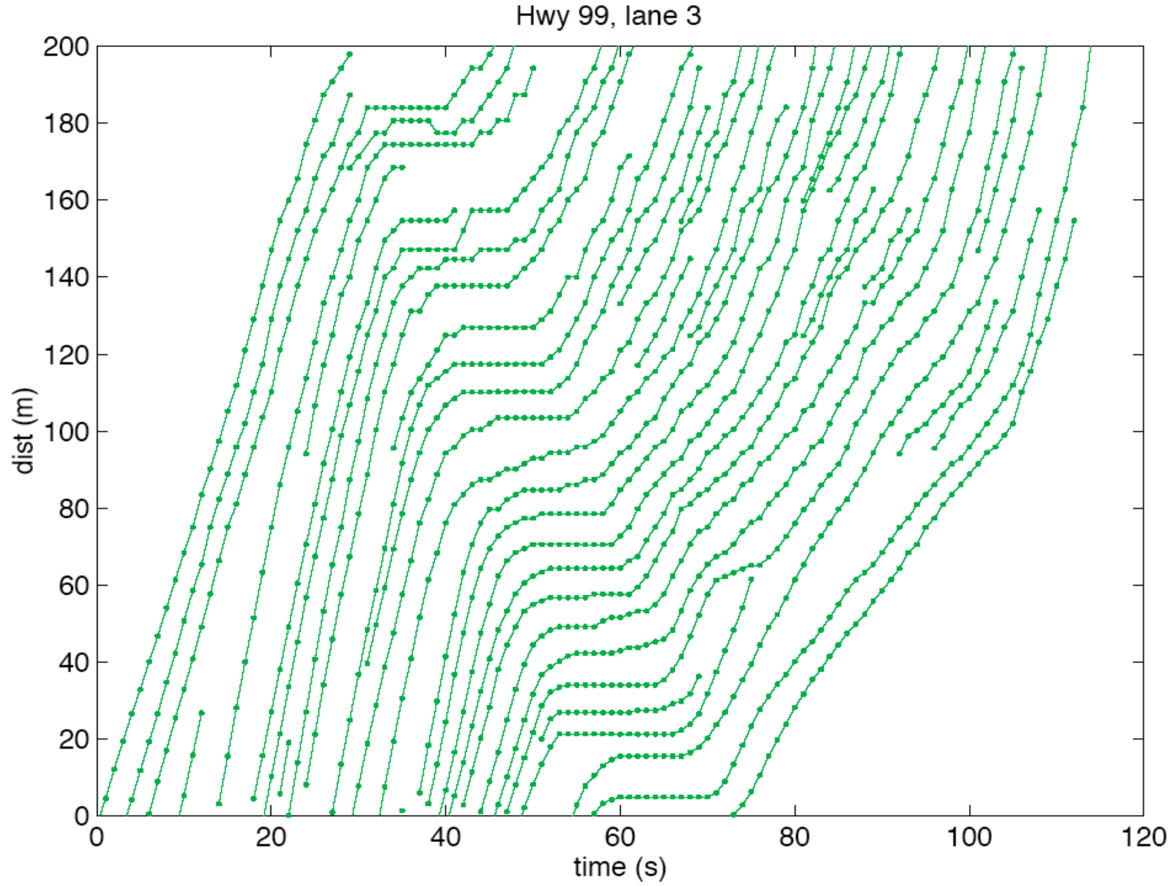

FIG. 1: $x - t$ space.

(a) Find P as a function of μ and ϵ.

(b) With $\epsilon = \epsilon_0 + \rho_0 u^2/2$ solve Eq. (3) for u.

(c) Find P/ϵ_0 as a function of ρ/ρ_0.

(d) Assuming that the *corrections* are small show how this EOS differs from the ideal gas EOS.