P740.HW1.sol.tex

1. Speed of tsunamis. This is just about numbers. With plausible choices the depth of an earth ocean is a few kilometers.

2. Size of linear fluctuations. See note **P740.1.tex**. The important physical idea is that all of the dimensionless measures of the sizes of things are about the same. From below Eq. (4) find the relationship of $\delta\rho$ to δP. Put it in the form

$$\frac{\delta\rho}{\rho_0} = \frac{P_0}{\rho_0}\left(\frac{\partial\rho}{\partial P}\right)\frac{\delta P}{P_0} = C_{\rho,P}\frac{\delta P}{P_0}. \tag{1}$$

From Eq. (9)

$$\frac{\delta\rho}{\rho_0} = \frac{|\delta\mathbf{v}|}{c_0} = C_{\rho,P}\frac{\delta P}{P_0}. \tag{2}$$

For an ideal gas $(\partial\rho/\partial P) = \rho/P$, i.e., $C_{\rho,P} = 1$ and

$$\frac{\delta\rho}{\rho_0} = \frac{|\delta\mathbf{v}|}{c_0} = \frac{\delta P}{P_0}. \tag{3}$$

Even when you don't have an ideal gas you might begin by expecting this relation to be approximately the case.

There is a second point, the astonishing *dynamic range* of the ear. Look up some numbers.

3. Random walker. Examples of MATLAB files for doing this problem are on the course web page, Homework, **rwlistings**.

This problem can also be done analytically. From

$$(a+b)^N = \sum_{n=0}^{N} a^{N-n}b^n \frac{N!}{n!(N-n)!} = \sum_{n=0}^{N} p_n, \tag{4}$$

where $a + b = 1$ and p_n is the probability of taking $N - n$ steps to the right and n steps to the left. When the walker has taken n steps to the left he is at $L_n = N - 2n$ (a factor of 2 because he didn't take the n steps to the right). The average distance (squared distance) the walker goes in N steps is

$$< L > \;=\; \sum_{n=0}^{N} (N-2n)p_n = N - 2 < n >, \tag{5}$$

$$
\begin{aligned}
< L^2 > &= \sum_{n=0}^{N} (N-2n)^2 p_n = N^2 - 4N < n > +4 < n >^2, && (6) \\
< f(n) > &= \sum_{n=0}^{N} f(n)\; p_n. && (7)
\end{aligned}
$$

If you write out the equations for $< n >$ and $< n^2 >$ you can show that

$$
\begin{aligned}
< n > &= b\frac{d}{db}\sum_{n=0}^{N} p_n = b\frac{d}{db}(a+b)^N = Nb, && (8) \\
< n^2 > &= b\frac{d}{db}b\frac{d}{db}\sum_{n=0}^{N} p_n = b\frac{d}{db}b\frac{d}{db}(a+b)^N = N^2b^2 + Nb(1-b). && (9) \\
& && (10)
\end{aligned}
$$

etc. Thus

$$
\delta L^2 = < L^2 > - < L >^2 = 4Nb(1-b) \tag{11}
$$

for any b; $< L^2 > - < L >^2 = N$ for $b = 1/2$, i.e. $< x^2 >= D_D t$ with $D_D = a^2/\tau$ (subscript D for *discrete*).

Aside. When you study the RW problem in the continuum approximation

$$
\frac{\partial p}{\partial t} = D_C \frac{\partial^2 p}{\partial x^2} \tag{12}
$$

(subscript C for *continuum*) you get

$$
< x^2 >= 2D_C t. \tag{13}
$$

The apparent discrepancy is resolved when you turn the discrete RW into a continuum RW. You find $D_C = D_D/2$.