



Lossy image compression using singular value decomposition and wavelet difference reduction

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ABSTRACT

This paper presents a new lossy image compression technique which uses singular value decomposition (SVD) and wavelet difference reduction (WDR). These two techniques are combined in order for the SVD compression to boost the performance of the WDR compression. SVD compression offers very high image quality but low compression ratios; on the other hand, WDR compression offers high compression. In the Proposed technique, an input image is first compressed using SVD and then compressed again using WDR. The WDR technique is further used to obtain the required compression ratio of the overall system. The proposed image compression technique was tested on several test images and the result compared with those of WDR and JPEG2000. The quantitative and visual results are showing the superiority of the proposed compression technique over the aforementioned compression techniques. The PSNR at compression ratio of 80:1 for Goldhill is 33.37 dB for the proposed technique which is 5.68 dB and 5.65 dB higher than JPEG2000 and WDR techniques respectively.

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1. Introduction

Most applications in human lives such as medicine, ecommerce, astronomy, and remote sensing deal with enormous amounts of digital images [1–5]. This has led to the sharing and storage of large amounts of digital images. The amount of data needed to represent digital images makes transmission slow and storage expensive. The amount of data used to represent these images therefore needs to be reduced.

Image compression deals with reducing the number of bits needed to represent an image by removing redundant data. Psychovisual redundancy takes advantage of the fact that the human eyes ignore some data [6], coding redundancy uses codewords to represent the statistics of the original data while interpixel redundancy explores the fact that some pixels in an image have the same or almost the same value [7,8]. Image compression is broadly classified into two categories, namely lossless and lossy, depending on whether the original image can be recovered with full mathematical precision from the compressed image [7]. In lossless techniques, the original image can be recovered perfectly from the compressed image [8]. In lossy techniques, the original image cannot be recovered from the compressed image as some quantization losses are encountered during the encoding of the image [9–12].

JPEG2000 is a high performance image compression technique developed by the Joint Photographic Experts Group committee. JPEG2000 is based on the discrete wavelet transform and also uses ‘tiling’ which refers to the partitioning of the original image into rectangular non-overlapping blocks (tiles), which are compressed independently, as though they were entirely distinct images [13,14]. Tiling reduces memory requirements and also improves the compression performance which makes JPEG2000 a state of the art image compression technique. However, other simple algorithms like the wavelet difference reduction (WDR) algorithm can be used as an alternative to JPEG2000 while achieving comparable performance [15].

In this paper a new image compression technique which uses singular value decomposition (SVD) and WDR compression techniques is proposed. SVD is a lossy compression technique which achieves compression by using a smaller rank to approximate the original matrix representing an image [16]. The WDR is also compression technique which is based on the wavelet difference algorithm [17–20]. In our proposed technique, the SVD and WDR methods were combined to make a lossy image compression. SVD compression technique is combined with WDR compression to complement each other. This is because SVD compression technique offers very good PSNR values but very low compression ratios [8,21,23–26]. On the other hand, WDR compression technique offers very good PSNR values at very high compression ratios. However, the performance of the WDR technique can be improved by combining WDR technique with SVD image compression

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Fig. 1. (a) Original Lena image; Lena's image of size 256×256 reconstructed by Eq. (4) using (b) 128 singular values, (c) 64 singular values and (d) 32 singular values.

technique. The proposed image compression technique has been tested on well-known images like, Airfield, Boats, Goldhill, Lena, and Peppers and has been compared with the JPEG2000 and WDR techniques.

The quantitative experimental results based on PSNR show that the proposed technique outperforms the above mentioned techniques. This paper is organized as follows: In the second section, we discuss the overview of the methodologies used in our proposed image compression technique in which SVD and WDR are given in details. Then in the third section the proposed lossy image compression is introduced and discussed in details. The experimental results and discussions are given in Section 4 followed by conclusion in the last section.

2. Overview of SVD and WDR

2.1. Singular value decomposition

An image is actually a matrix of numbers whose elements are the intensity value of corresponding pixels of the image. Singular value decomposition is used in order to decompose a given matrix into three matrices known as, U , Σ , and V in which U and V are orthogonal and Σ is a diagonal matrix containing the sorted singular values of the input matrix in descending order [21,22]. Eq. (1) is showing the size of the U , Σ , and V matrices for a given $m \times n$ input matrix.

$$A_{m \times n} = U_{m \times m} \Sigma_{m \times n} (V_{n \times n})^T \quad (1)$$

The number of non-zero elements on the diagonal of Σ determines the rank of the input matrix. Compression is done by using a smaller rank Σ obtained by eliminating small singular values (σ_i) to approximate the original matrix. Mathematically it can be describe as follows:

$$\Sigma_{m \times n} = \begin{bmatrix} \bar{\Sigma}_{p \times q} & 0 \\ 0 & \ddots \end{bmatrix} \quad p \leq m \text{ and } q \leq n \quad (2)$$

As $\bar{\Sigma}$ has less row and column with respect to Σ thus some column of U and rows of V need to be reduced in order to be able to conduct the matrix multiplications for reconstructing the image, as shown in Eq. (3):

$$U_{m \times m} = [\bar{U}_{m \times p} \quad \bar{U}_{m \times (m-p)}] \quad \text{and} \quad V_{n \times n} = [\bar{V}_{n \times q} \quad \bar{V}_{n \times (n-q)}] \quad (3)$$

Hence the reconstructed matrix can be obtained by

$$A_{m \times n} = \bar{U}_{m \times p} \bar{\Sigma}_{p \times q} (\bar{V}_{n \times q})^T \quad (4)$$

Because the singular matrix has sorted singular values (in descending order) by using the psychovisual concept, ignoring low singular value will not significantly reduce the visual quality of the image. Fig. 1 is showing Lena's picture being reconstructed by using different amount of singular values.

This characteristic that an image can be reconstructed by fewer amounts of singular values takes SVD suitable for compression. Because after reconstruction of the image the ignored singular values cannot be recovered, the compression by SVD is lossy.

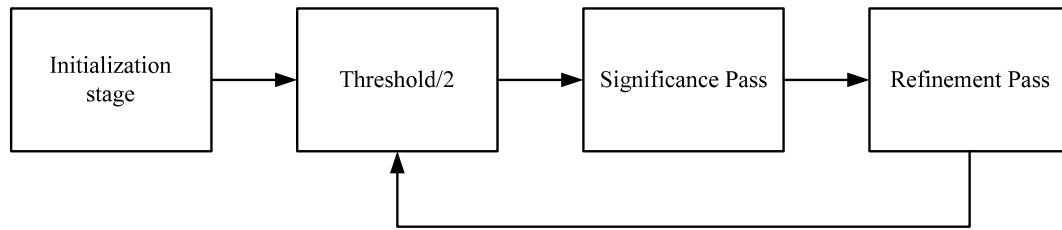


Fig. 2. Block diagram of Bit-Plane encoding used by WDR [11].

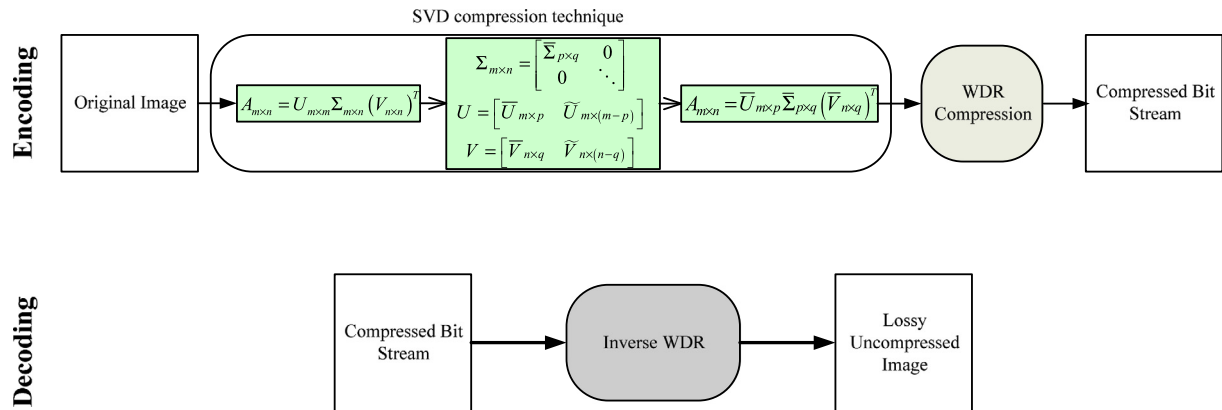


Fig. 3. Block diagram of the proposed image compression technique.

2.2. Wave difference reduction

The Wavelet Difference Reduction (WDR) is a simple compression method which can be lossy or lossless. In this work lossy WDR is used. A wavelet transform (DWT) is applied to the image followed by a bit plane encoding procedure for the transform values. This bit plane encoding procedure consists of five procedures as shown in Fig. 2. At the initialization stage, an initial threshold T_0 is chosen such that the value of T_0 is greater than all the transform values and at least one transform value has a magnitude of $T_0/2$. After the initialization stage, the threshold $T = T_{k-1}$ is updated to $T = T_k$, where $T_k = T_{k-1}/2$. At the significant pass stage, new significant transform values ($w(i)$) which satisfy $T \leq |w(i)| \leq 2T$ are identified. Their index values are then encoded using the difference reduction method [8,9].

As an example of WDR consider a threshold $T_1 = 32$, with significant values $w(1) = 63$, $w(2) = -34$, $w(5) = 49$, and $w(36) = 47$. The indices for these significant values are 1, 2, 5 and 36. Rather than working with these values, WDR works with their successive differences: 1, 1, 3, and 31. In this latter list, the first number is the starting index, and each successive number is the number of steps needed to reach the next index. The binary expansions of these successive differences are $(1)_2$, $(1)_2$, $(11)_2$, and $(11111)_2$. Since the most significant bit for each of these expansions is always 1, this bit can be dropped and the signs of the significant transform values can be used instead as separators in the symbol stream. The output of the WDR significance pass will then be the following string of symbols [12]:

$$+ - +1 + 1111 \quad (5)$$

At the refinement pass stage, error is reduced by refining the already quantized values (w_Q) which satisfy $|w_Q| \geq 2T$. Each refined value is a better approximation of the original transform value. For example, if an old significant transform value's magnitude $|w_n|$ lies in the interval $[64; 128)$, say, and the present threshold is 32, then it will be determined if its magnitude lies in $[64; 96)$ or $[96; 128)$. In the latter case, the new quantized value

becomes $96 \operatorname{sgn}(w_n)$, and in the former case, the quantized value remains $64 \operatorname{sgn}(w_n)$ [11].

In order to decompress an image, the above steps are reversed to obtain an approximate wavelet transform and then the inverse DWT (IDWT) is taken to obtain the decompressed image.

3. The proposed lossy image compression technique

As it was mentioned in the previous section, the SVD based compression is lossy due to the nature of the process. However, the qualitative loss is not noticeable up to some point, as shown in the previous section. The proposed compression technique is benefiting from cascading SVD based compression followed by WDR based compression. This is due to the fact that SVD compression technique offers very good PSNR values but low compression ratios and WDR compression technique offers high compression ratios. The compression ratio of SVD based compression technique is obtained by calculating the required number of bits to represent U , Σ , and V over the required number of bit to represent $\bar{U}_{m \times p}$, $\bar{\Sigma}_{p \times q}$, and $\bar{V}_{q \times n}$. First the image is being compressed by using SVD as shown in Eq. (4). The reconstructed image by using Eq. (4) is then further compressed by using WDR.

The image is first decomposed using SVD, some singular values are ignored, and then the image is reconstructed. The reconstructed image is then used as the input image for the WDR part of the proposed technique. At the WDR side of the technique, maximum number of loops for the WDR technique is fixed at 12. The higher the number of singular values ignored at the SVD part of the proposed technique, the higher the compression ratio. The overall compression ratio is then calculated as the multiplication of the SVD based compression with that of WDR. The block diagram of the proposed compression technique is shown in Fig. 3.

In the next section the experimental results are reported. The results are clearly showing the superiority of the proposed lossy image compression technique over those of JPEG2000 and WDR technique.

Table 1
PSNR values in dB for 20:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	27.02	27.32	32.26
Artificial	29.46	32.36	33.63
Big_building	27.20	29.01	27.97
Boats	32.42	33.18	33.81
Bridge	26.16	28.24	33.31
Deer	37.76	41.05	40.36
Fireworks	32.60	36.38	42.06
Goldhill	31.76	32.18	37.77
Lena	35.72	35.99	39.05
Peppers	34.21	35.07	39.12

Table 2
PSNR values in dB for 40:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	24.72	24.88	30.35
Artificial	26.44	28.58	31.31
Big_building	24.89	26.54	28.83
Boats	29.32	29.76	31.47
Bridge	24.16	25.21	31.19
Deer	34.00	36.34	35.87
Fireworks	28.71	30.97	38.76
Goldhill	29.43	29.72	36.72
Lena	32.44	32.75	36.58
Peppers	31.67	32.40	36.91

Table 3
PSNR values in dB for 80:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	22.71	22.64	25.50
Artificial	23.82	25.69	26.69
Big_building	22.73	24.33	24.84
Boats	26.96	26.76	27.56
Bridge	22.03	23.03	27.21
Deer	30.75	31.96	30.44
Fireworks	27.07	27.30	33.03
Goldhill	27.72	27.69	33.37
Lena	29.71	29.62	31.24
Peppers	28.93	29.54	31.90

Table 4
SSIM comparison for 20:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	0.620	0.698	0.897
Artificial	0.830	0.897	0.977
Big_building	0.715	0.786	0.911
Boats	0.844	0.898	0.934
Bridge	0.812	0.877	0.957
Deer	0.954	0.969	0.974
Fireworks	0.882	0.897	0.965
Goldhill	0.766	0.839	0.959
Lena	0.874	0.910	0.968
Peppers	0.842	0.875	0.967

Table 5
SSIM comparison for 40:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	0.548	0.606	0.873
Artificial	0.734	0.737	0.967
Big_building	0.620	0.703	0.895
Boats	0.767	0.828	0.904
Bridge	0.739	0.792	0.940
Deer	0.930	0.948	0.952
Fireworks	0.814	0.850	0.948
Goldhill	0.695	0.752	0.949
Lena	0.825	0.868	0.956
Peppers	0.795	0.839	0.960

Table 6
SSIM comparison for 80:1 compression.

Image/Method	WDR	JPEG2000	Proposed
Airfield	0.470	0.520	0.752
Artificial	0.620	0.767	0.910
Big_building	0.521	0.596	0.832
Boats	0.710	0.740	0.809
Bridge	0.641	0.692	0.854
Deer	0.900	0.912	0.880
Fireworks	0.764	0.606	0.898
Goldhill	0.625	0.670	0.899
Lena	0.770	0.808	0.883
Peppers	0.743	0.783	0.907

4. Experimental results and discussion

As it was mentioned in the introduction, the proposed lossy image compression was tested on Airfield, Boats, Goldhill, Lena and Peppers images. Table 1 is showing the quantitative comparison between the proposed techniques and JPEG2000 and WDR by use of PSNR for compression ratio of 20:1. In order to see the performance of the proposed image compression technique with different compression ratios Tables 2 and 3 are with compression ratio of 40:1 and 80:1 respectively are prepared. All the images used are 512×512 with 8-bit grey-scale representation. The PSNR values of the JPEG2000 compression were obtained from [10]. In order to ensure consistency, the same test images used in [10] were used.

As the PSNR values show in Tables 1, 2 and 3 the performance of the proposed technique overcomes the JPEG2000, and WDR image compression techniques. As an example, the PSNR value of the proposed compression technique for Goldhill image is 5.59 dB, 7.00 dB, and 5.65 dB higher than that of JPEG2000 compression technique for 20:1, 40:1, and 80:1 compression ratios respectively.

Tables 4, 5 and 6 show the comparison of the SSIM values for the proposed technique with those of WDR and JPEG2000. The SSIM values show that the proposed technique outperforms both WDR and JPEG2000 techniques.

Fig. 4 is illustrating PSNR versus the compression ratio for Peppers image comparing the state-of-the-art techniques used in the

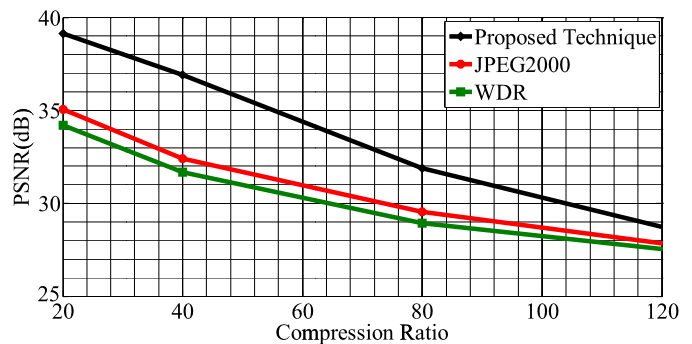


Fig. 4. Comparison of PSNR values (dB) of peppers image for different compression ratios for the proposed technique, JPEG2000 and WDR technique.

work and our proposed technique. The graphs are showing the superiority of the proposed image compression over the other state-of-the-art techniques.

Fig. 5 and Fig. 6 show the bridge and deer images compressed at a compression ratio of 40:1 using JPEG2000, WDR and the proposed technique.

Fig. 7 is shows the compressed peppers image at various compression ratios of the proposed technique. It compares the visual quality of the original pepper image with those at compression ratios of 20:1, 40:1, 60:1, 80:1, and 160:1. Fig. 7 also shows that

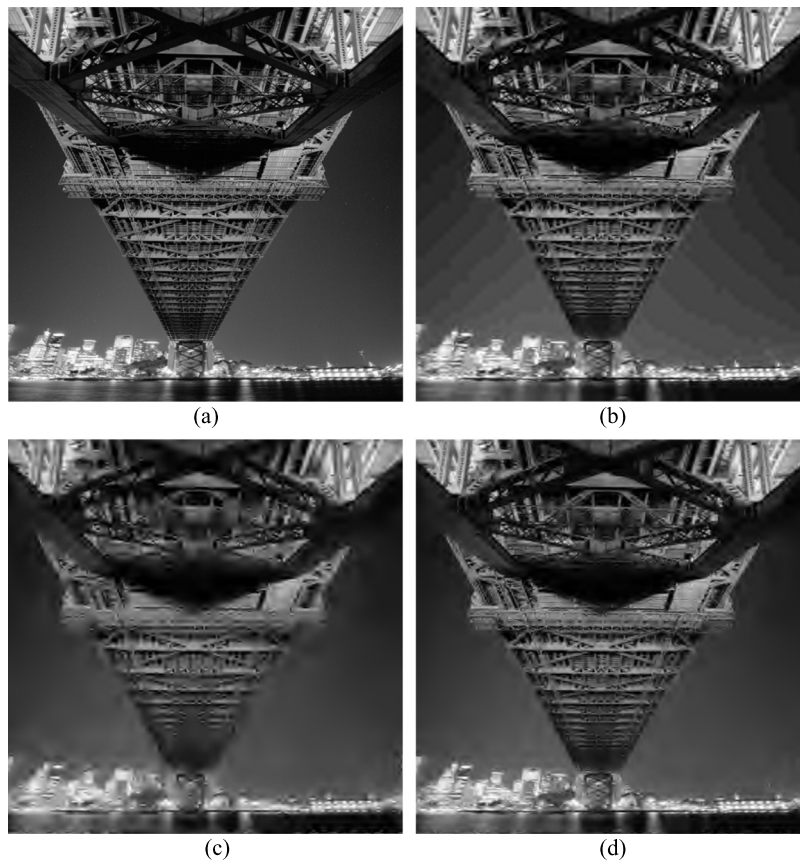


Fig. 5. (a) Original bridge image, and compressed image by using (b) JPEG2000, (c) WDR, and (d) the proposed image compression technique at 40:1 compression ratio.

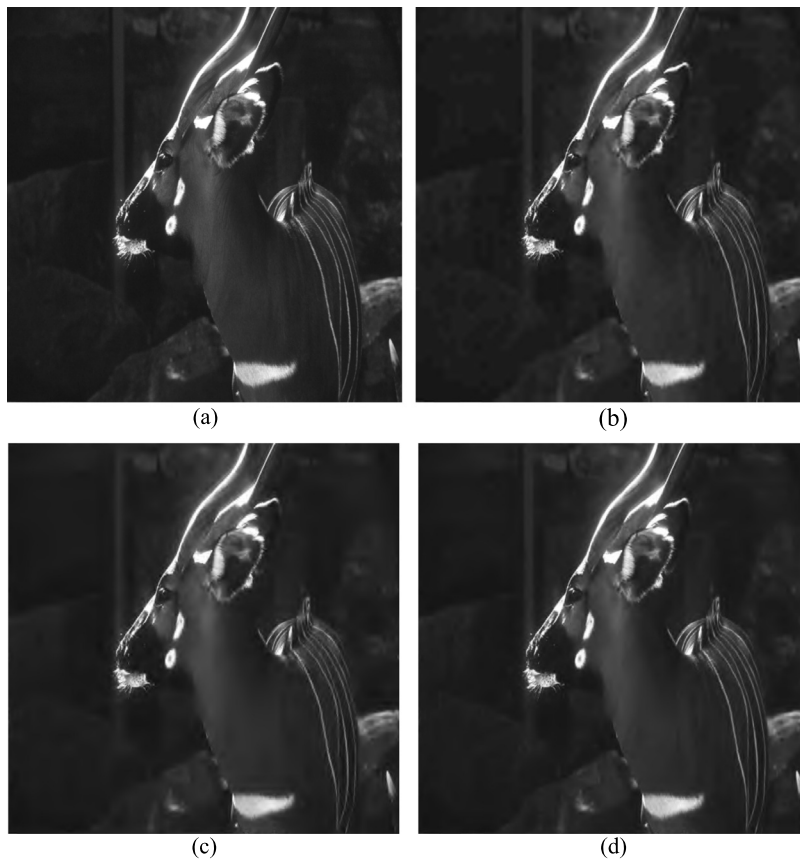


Fig. 6. (a) Original deer image, and compressed image by using (b) JPEG2000, (c) WDR, and (d) the proposed image compression technique at 40:1 compression ratio.

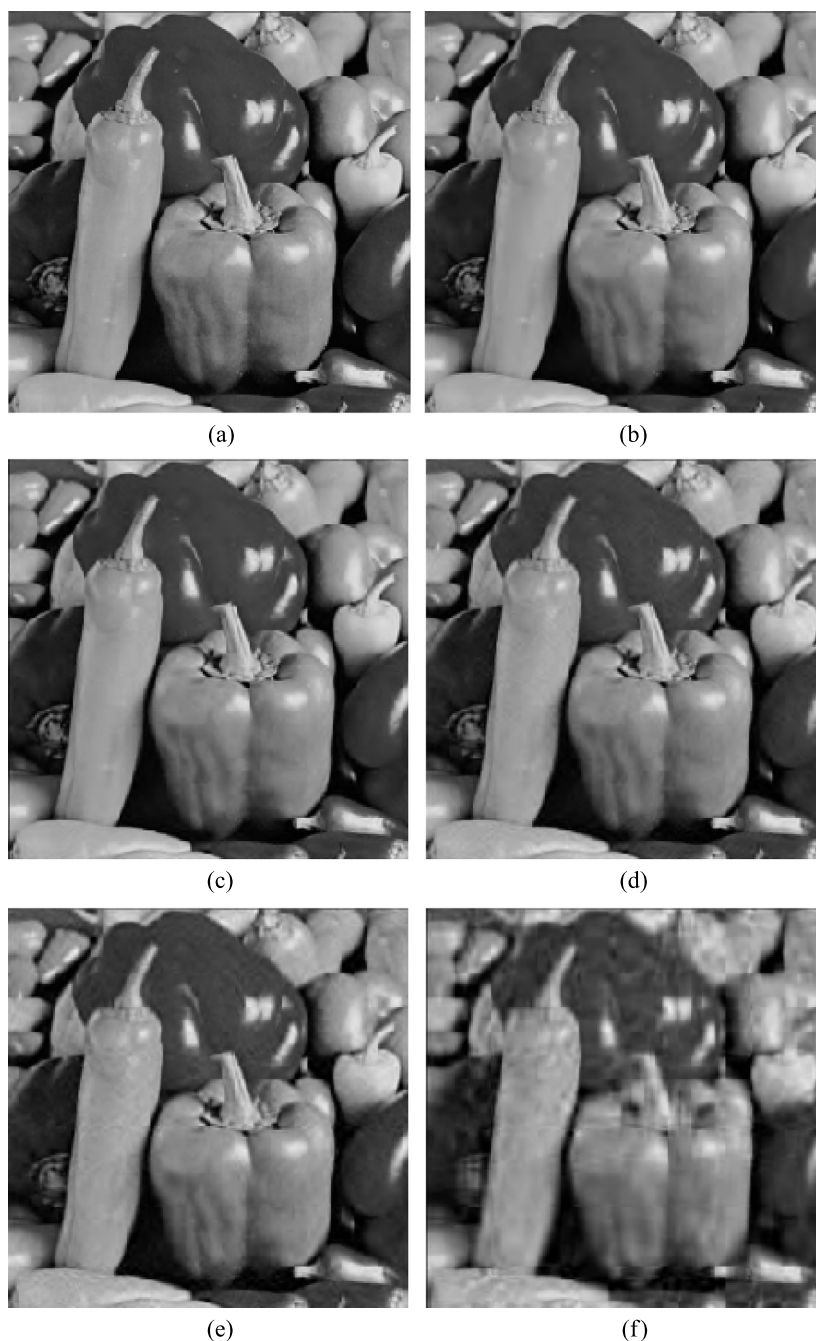


Fig. 7. (a) Original uncompressed Pepper image, (b) Pepper image at compression ratio of 20:1, (c) Pepper image at compression ratio of 40:1, (d) Pepper image at compression ratio of 60:1, (e) Pepper image at compression ratio of 80:1, (f) Pepper image at compression ratio of 160:1.

disturbing noise due to loss of more important singular values in compression by SVD can only be noticed at 160:1 compression ratio.

Fig. 8 is illustrating the relationship between PSNR values in dB and compression ratio with respect to the selected number of singular values in proposed image compression technique for peppers image whose size is 512×512 .

In general, the quantitative and qualitative results show that the proposed lossy compression technique is an outstanding technique with high performance even at high compression ratios. The reason for the high gain in PSNR values is due to the fact that SVD compression serves as a 'booster' as it compresses the image without much loss in quality and therefore boosting the overall compression when WDR compression is applied.

5. Conclusion

In this paper, we have proposed a new lossy image compression technique by using SVD and WDR. The proposed technique was using SVD in order to neglect the low singular values, and then reconstruct a compressed image. This image was compressed again by using WDR. The compression ratio was obtained by multiplication of the SVD based compression ratio with the WDR based compression ratio. The results of the proposed technique were also compared with several state-of-the-art image compression techniques. The quantitative and visual results showed the superiority of the proposed compression technique over the state-of-the-art techniques.

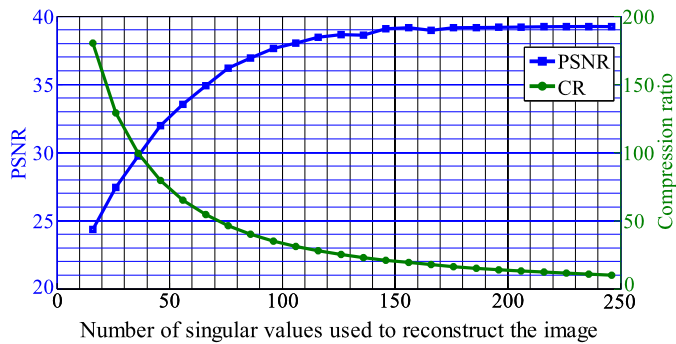


Fig. 8. Graph showing PSNR, compression ratio and number of singular values used for peppers image.

References

- [1] T. Glatard, C. Lartizien, B. Gibaud, R.F. da Silva, G. Forestier, F. Cervenansky, M. Alessandrini, H. Benoit-Cattin, O. Bernard, S. Camarasu-Pop, N. Cerezo, P. Clarysse, A. Gagnard, P. Hugonnard, H. Liebgott, S. Marache, A. Marion, J. Montagnat, J. Tabary, D. Friboulet, A virtual imaging platform for multi-modality medical image simulation, *IEEE Trans. Med. Imag.* 32 (2013) 110–118.
- [2] H. Gu, G. Zhao, J. Qiu, Online metric learning for relevance feedback in e-commerce image retrieval applications, *Tsinghua Sci. Technol.* (2011).
- [3] F. García-Vilchez, J. Muñoz-Marí, M. Zortea, I. Blanes, V. González-Ruiz, G. Camps-Valls, A. Plaza, J. Serra-Sagristà, On the impact of lossy compression on hyperspectral image classification and unmixing, *IEEE Geosci. Remote Sens. Lett.* 8 (2011) 253–257.
- [4] Ping Li, Xia-xin Tao, Jin-quan Zhang, Xin-zheng Wang, Retrieval module to choose satellite images by considering the demand of disaster mitigation, in: *International Conference on Remote Sensing, Environment and Transportation Engineering*, 2011, pp. 685–688.
- [5] M. Naixia, L. Bing, L. Xiushan, Z. Lingxian, L. Wenbao, Semantic-based remote sensing images intelligent service on grid environment, in: *First International Workshop on Database Technology and Applications*, 2009, pp. 291–294.
- [6] R.C. Gonzalez, R.E. Woods, *Digital Image Processing*, 3rd edition, Prentice-Hall, 2008.
- [7] M. Groach, A. Garg, DCSPHT: Image compression algorithm, *Int. J. Eng. Res. Appl.* 2 (2012) 560–567.
- [8] A.M. Rufai, G. Anbarjafari, H. Demirel, Lossy medical image compression using Huffman coding and singular value decomposition, in: *IEEE Signal Processing and Communications Applications Conference*, 2013, pp. 1–4.
- [9] A. Zabala, X. Pons, Impact of lossy compression on mapping crop areas from remote sensing, *Int. J. Remote Sens.* 34 (2013) 2796–2813.
- [10] N.N. Ponomarenko, V.V. Lukin, K.O. Egiazarian, L. Lepisto, Adaptive visually lossless JPEG-based color image compression, *J. Signal Image Video Process.* 7 (2013) 437–445.
- [11] X. Zhang, Lossy compression and iterative reconstruction for encrypted image, *IEEE Trans. Inf. Forensics Secur.* 6 (2011) 53–58.
- [12] H. Singh, S. Sharma, Hybrid image compression using DWT, DCT & Huffman encoding techniques, *Int. J. Emerging Technol. Adv. Eng.* 2 (2012) 300–306.
- [13] M. Charrier, D.S. Cruz, M. Larsson, JPEG2000, the next millennium compression standard for still images, in: *IEEE International Conference on Multimedia Computing and Systems*, vol. 1, 1999, pp. 131–132.
- [14] A.N. Skodras, T. Ebrahimi, JPEG2000 image coding system theory and application, in: *IEEE International Symposium on Circuits and Systems*, 2006, pp. 3866–3869.
- [15] J.S. Walker, Y. Chen, T.M. Elgindi, Comparison of the JPEG2000 lossy image compression algorithm with WDR-based algorithms, in: *IEEE International Conference on Image Processing*, 2006.
- [16] D. Kalman, A singularly valuable decomposition, *College Math. J.* 27 (1998) 2–23.
- [17] J.S. Walker, T.Q. Nguyen, Y. Chen, A low-power, low-memory system for wavelet-based image compression, *Opt. Eng., Res. Signposts* 5 (2003) 111–125.
- [18] J.S. Walker, T.Q. Nguyen, Wavelet-based image compression, chap. 6, in: K.R. Rao, P. Yip (Eds.), *Handbook of Transforms and Data Compression*, CRC Press, Boca Raton, 2001, pp. 267–312.
- [19] S.P. Raja, A. Suruliandi, Image compression using WDR and ASWDR techniques with different wavelet codecs, *ACEEE Int. J. Inform. Technol.* 01 (2011) 23–26.
- [20] Y. Yuan, M.K. Mandal, Novel embedded image coding algorithms based on wavelet difference reduction, *IEE Proc. Vision Image Signal Process.* 152 (2005) 9–19.
- [21] J.F. Yang, C.L. Lu, Combined techniques of singular value decomposition and vector quantization for image coding, *IEEE Trans. Image Process.* 4 (8) (1995) 1141–1146.
- [22] J. Tian, R.O. Wells Jr., A lossy image codec based on index coding, in: *IEEE Data Compression Conference*, 1996, pp. 456–468.
- [23] J. Tian, R.O. Wells Jr., Embedded image coding using wavelet difference-reduction, in: P. Topiwala (Ed.), *Wavelet Image and Video Compression*, Kluwer Academic Publ., Norwell, MA, 1998, pp. 289–301.
- [24] J. Tian, R.O. Wells Jr., Image data processing in the compressed wavelet domain, in: B. Yuan, X. Tang (Eds.), *3rd International Conference on Signal Processing Proc.*, 1996, pp. 978–981.
- [25] M. Greenberg, *Differential Equations & Linear Algebra*, 1st edition, Prentice-Hall, 2001.
- [26] S.J. Leon, *Linear Algebra with Applications*, 4th edition, Macmillan, New York, 1994.

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