Physics 740: Spring 2006:

Afternote.1.tex 01/25/07

Note on Diffusion and $\partial P/\partial \rho$.

A. The diffusion equation is

$$\frac{\partial \rho_A}{\partial t} = D_A \nabla^2 \rho_A, \tag{1}$$

where ρ_A is the density of A and D_A is the diffusion constant of A. In the Navier-Stokes equation we have

$$\frac{\partial \mathbf{v}}{\partial t} + \cdots = \cdots + D_\eta \nabla^2 \mathbf{v}. \tag{2}$$

If all the stuff not shown were unimportant the velocity field, $\mathbf{v}$, would diffuse. Since the diffusion equation is of the form

$$\frac{\partial}{\partial t} \; = \; D\nabla^2, \tag{3}$$

the units of D are $[D] = L^2/T$.

B. If you have a theory of D you usually find $D \approx vl$ where v is a velocity and l a mean free path. We'll find some results like this in a few days. There ia a nice way to re-write the equation for D and re-think what it means. A particle with velocity v will travel distance l in time $\tau = l/v$. So

$$D \approx \frac{l^2}{\tau}. \tag{4}$$

Use l and τ to describe D; note the right units.
The way to think about $D \approx l^2/\tau$ is that it describes a random walker who

1. takes random steps of size l, e.g., $\pm l$ in d=1,
2. every τ seconds (you can think that he takes τ seconds to take the step or that he takes the step instantly and waits τ seconds before taking the next step or ...).

C. When you solve Eq. (1) for diffusion in d=1 you find results that lead to

$$< x > \; = \; 0, \tag{5}$$

$$< x^2 > \; \propto \; Dt. \tag{6}$$

The first result is obvious, if you take steps $\pm l$ equally probably you will on average be nowhere, i.e., at $x = 0$. The second result says that you make progress on average more slowly than ballistic flight for which $x \propto t$ (in ballistic flight each step is deliberately in the direction of the last, a very biased random walk?). In fact $\overline{x} = \sqrt{< x^2 >} \propto \sqrt{t}$. It is these results that lead to the discussion in class about the approximate time for diffusion of density to wipe out a sound wave.

1. a sound wave of wavelength λ causes a density fluctuation over length λ that hangs around for about one period, T_0, before moving on.

2. if the diffusion process can wipe out the density fluctuation it will cause decay of the sound wave (attenuation). To wipe out the density fluctuation particles must diffuse a distance of about λ. The time to do this is

$$t_\lambda = \lambda^2/D \tag{7}$$

   from Eq. (6). If $t_\lambda < T_0$ the diffusion process will be successful.

3. the degree to which the diffusion process is successful is measured by the ratio T_0/t_λ. This is the quantity we found in class for the strength of the attenuation of sound. Compare to appropriate arrangement of Eq. (13) in Note **P740.1.tex**

D. We needed a thermodynamic derivative to close the system of equations we were dealing with, i.e., we replace δP by $\delta\rho$ by appealing to an equation of state, $P = P(\rho)$ and

$$\delta P = \left(\frac{\partial P}{\partial \rho}\right)_X \delta\rho. \tag{8}$$

What is X? When Newton calculated the speed of sound he took $X = T$ and got scolded for 3 Centuries. How do you know what to do?