P740.HW4.sol.tex

1. n-star = pulsar.

1. The mass of the Sun is 2×10^{33} gm. Since $m_n \approx 2\times 10^{-24}$ gms, the Sun contains about $10^{57} = (10^{19})^3$ particles. So it is a cube with 10^{19} particles on a side. If the particles are spaced by 1 fermi = 10^{-13} cm, the cube has sides of length 10^6 cm = 10 km. A McCarran is π km so the n-star has approximate radius of 1 McC.

2. From conservation of angular momentum
$$\frac{T_\odot}{T_n} = \frac{\omega_n}{\omega_\odot} = \frac{R_\odot^2}{R_n^2} \approx 5\times 10^9. \tag{1}$$
The Sun's rotation rate is approximately 1/30 per day. So estimate $T_n \approx 0.5$ msec. That is fast for a n-star. There are lots of caveats. At the speed of light it would take the sun about 15 sec to get around once. So Sun-like objects in rotation are not good candidates for n-stars.

3. When r is scaled by R and ρ by M/R^3 the equation of hydrostatic equilibrium takes the form
$$\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2 y^{-\frac{1}{3}}\frac{\partial y}{\partial x}\right) = -Qy. \tag{2}$$
where
$$Q = 4\pi G R M^{\frac{1}{3}}\,\frac{m_n^{\frac{8}{3}}}{\hbar^2}. \tag{3}$$
Q is the ratio of the gravitational pressure $P_G = GM^2/R^4$ and the Pauli pressure
$$P_P = \frac{N}{V}\frac{\hbar^2}{m_n}\left(\frac{N}{V}\right)^{\frac{2}{3}} \approx \frac{\hbar^2}{m_n}\left(\frac{M}{m_n R^3}\right)^{\frac{5}{3}}, \tag{4}$$
upon using $N = M/m_n$ and $V \approx R^3$. Find $P_G \approx 3\times 10^{35}$ cgs and $P_P \approx 10\times 10^{35}$ cgs. The two pressures are of the same order of magnitude, roughly 10^{29-30} atmospheres!

4. Estimate $m_n c_N^2 \approx \hbar^2/(m_n a_N^2)$, $c_N \approx 2\times 10^9$ cm/sec. Vibration time of order 0.5 msec. Again the Sun is much too big. [The proposal that pulsars were n-stars used these types of arguments to eliminate most astrophysical objects as candidates for explaining the fast pulsing.]

2. vortices.

1. From inspection of the figures, e.g., lower left

$$x_1 = R_1\mathcal{S}, \tag{5}$$

$$y_1 = y_0 + R_1\mathcal{C}, \tag{6}$$

$$x_2 = R_2\mathcal{S}, \tag{7}$$

$$y_2 = y_0 + R_2\mathcal{C}, \tag{8}$$

where $\mathcal{S}, \mathcal{C}$ are $sin\omega t$ and $cos\omega t$, y_0 is the center of a circle centered on the $x = 0$ axis, $R_{1,2}$ are radii and ω is the frequency of the motion. Substitute this guess into the equations of motion and find, e.g., from the $\dot{x}_1$ and $\dot{x}_2$ equations

$$R_1 = 2K_2/(K_1 + K_2), \tag{9}$$

$$R_2 = -2K_1/(K_1 + K_2), \tag{10}$$

$$\omega = K_1 + K_2, \tag{11}$$

$$y_0 = (K_1 - K_2)/(K_1 + K_2). \tag{12}$$

Solve and find $R_1 = 4/3$, $R_2 = -2/3$, $\omega = 3$ and $y_0 = -1/3$. For the lower right find $R_1 = 4$, $R_2 = 2$, $\omega = -1$ and $y_0 = -3$.

3. quantized vortex profile?

Begin with the Euler equation in the form

$$-\frac{\kappa^2}{r^3}\mathbf{e}_r = -\frac{1}{\rho_0}\frac{\partial P}{\partial r}\mathbf{e}_r - \frac{1}{\rho_0}\frac{\partial P}{\partial z}\mathbf{e}_z - g\mathbf{e}_z, \tag{13}$$

where the term on the left side comes from

$$\mathbf{v} \cdot \nabla \mathbf{v} = \frac{\kappa}{r^2}\frac{\partial}{\partial \theta}\mathbf{v} = -\frac{\kappa^2}{r^3}\mathbf{e}_r \tag{14}$$

$\kappa = \hbar/m$. Find

$$P(r, z) = -\rho_0\frac{\kappa^2}{2r^2} - \rho_0 g z + C. \tag{15}$$

Fix the constant of integration by the choice at $z = 0$, $r \to +\infty$, $P = P_0$. Since P at the surface of the fluid is P_0, at the surface

$$z = -\frac{\kappa^2}{2gr^2}. \tag{16}$$

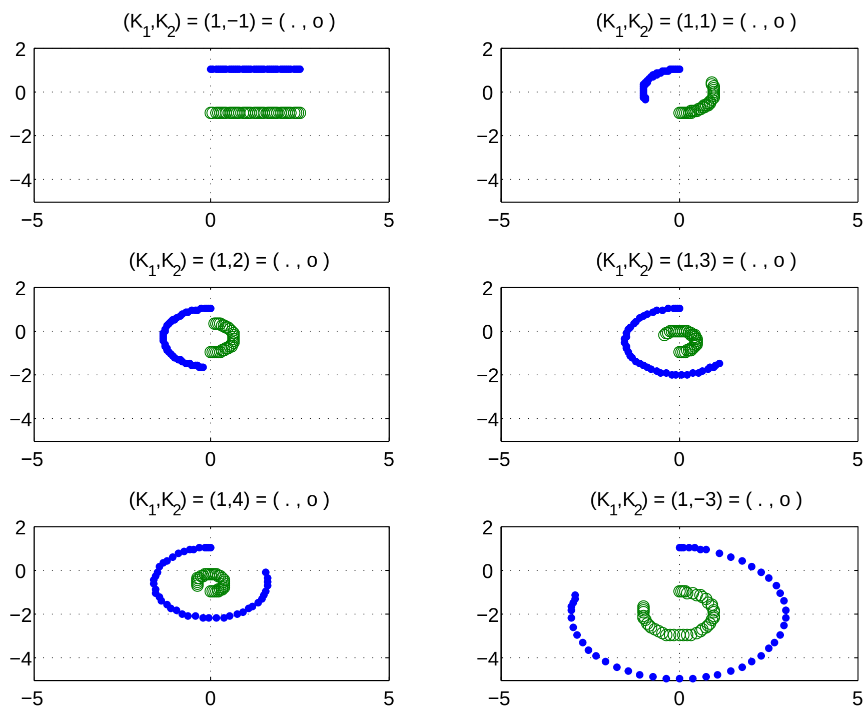

FIG. 1: From ode45 in MATLAB.

At $r \approx \sigma \approx 3$ Angstroms $z(a) \approx 100 - 200$ m. This is unphysical. What is missing? [As the quantization is Bohr quantization the speeds are, except for the mass, those of an electron in a Bohr orbit!] For gravity to produce the pressure gradient necessary to support the very rapid circular motion of the fluid at small radius the surface must be highly curved. [Bohr had $-e^2/r^2$.] This cost an enormous amount of surface energy which is not part of your calculation. All pieces of material would like to reduce the amount of surface they have? Why? Liquids can usually move to do so. In the case at hand a *proper* calculation would include the surface tension and give a very different result.

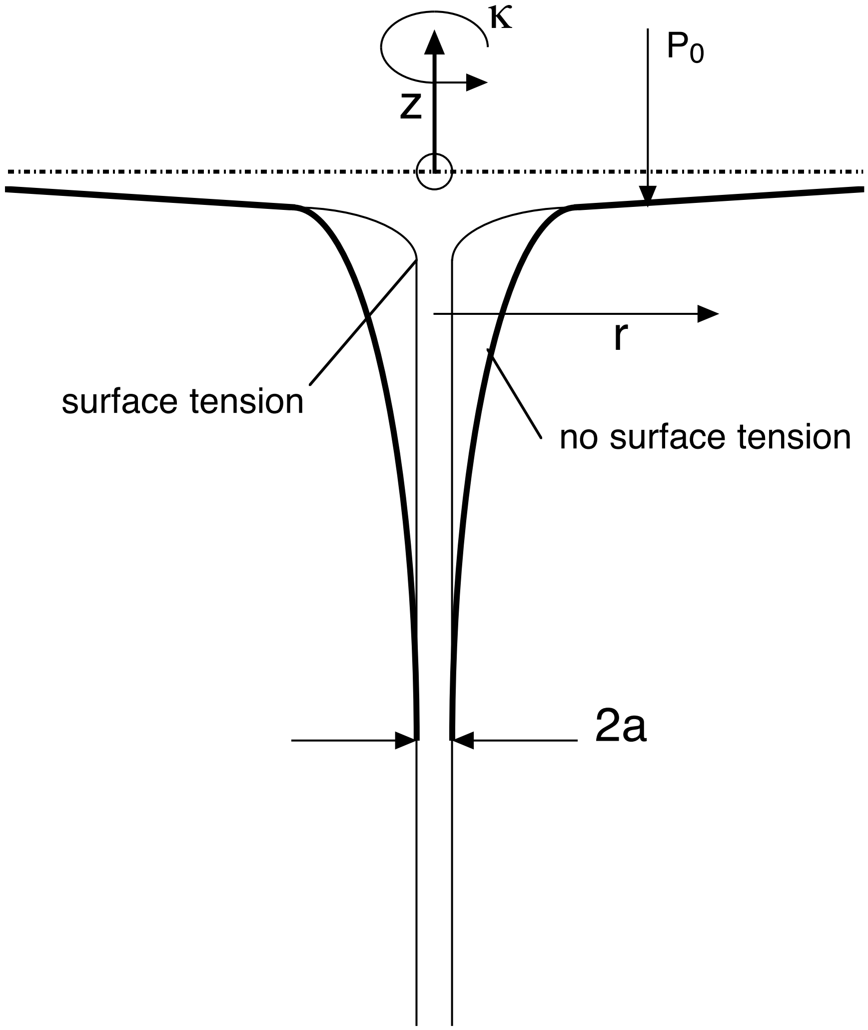

FIG. 2: Two fluid in a rotating tube.