P740.HW6.tex

Due 04/13/07

1. sine-Gordon.I Integrate the sine Gordon equation by quadrature, i.e.,

$$X^{''} = sinX \tag{1}$$

in the notation of Note 12. Find the solution for which $\phi = 0$ at $x \to -\infty$ and $\phi = 2\pi$ at $x \to +\infty$. Plot $\phi(x)$ vs x at $t = 0$.

2. KdV Integrate the KdV equation by quadrature.

3. KdV.II Variational procedure. Show that analytic variation of the energy functional in Eq. (20), Note 13, leads to the KdV equation, Eq. (5). Carry out the variational procedure for the KdV equation that is sketched at the end of Note 13.

4. sine-Gordon II. Look at Figs. 1 and 2. Take a pendulums on a torsion fiber point of view. Note 12. Let's agree by convention that a kink is an s-G soliton that twists the pendulums in accord with a right hand rule, i.e., with your thumb in the direction the kink is going your fingers wrap in the direction of the pendulum motion, that's a soliton. Fig. 1. It changes the angular coordinate of the pendulums from 0 to 2π. An anti-kink, going in the same direction, causes the pendulums to move counter-clockwise, by convention from 0 to -2π. These definitions are a convention, not necessarily universal. Consistency is all that is required.

1. What happens when two kinks, initially far apart, approach one another and collide?
2. What happens when a kink and anti-kink, initially far apart, approach one another and collide?

Look at wikipedia, sine-Gordon. And possibly elsewhere.

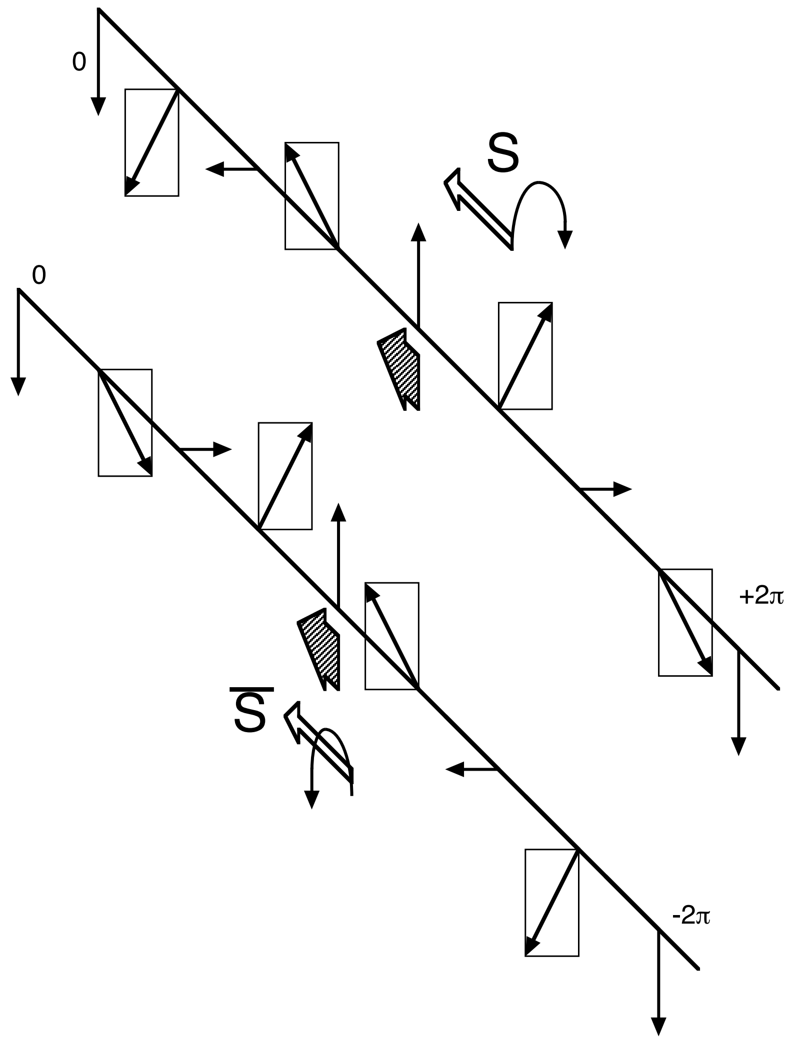

FIG. 1: Soliton and anti-soliton or kink and anti-kink.

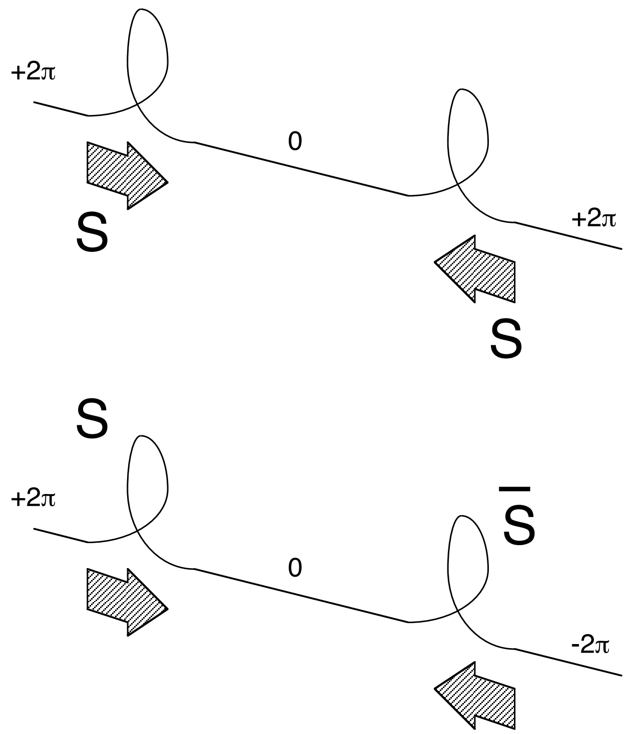

FIG. 2: Kink-kink collision and kink - anti-kink collision.