A mechanical model of a non-uniform Ionomeric Polymer Metal Composite (IPMC) actuator

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Abstract
This paper describes a mechanical model of an IPMC (Ionomeric Polymer Metal Composite) actuator in a cantilever beam configuration. The main contribution of our model is that it gives the most detailed description of the quasistatic mechanical behaviour of the actuator with a non uniform bending at large deflections reported so far. We also investigate a case where part of an IPMC actuator is replaced with a rigid elongation and demonstrate that this configuration would make the actuator to behave more linearly. The model is experimentally validated by with MuscleSheet™ IPMCs, purchased from BioMimetics Inc.

Subject classification numbers
81.05.Lg (Polymers and plastics; rubber; synthetic and natural fibers; organometallic and organic materials)
82.35.-x (Polymers: properties; reactions; polymerization)
82.35.Lr (Physical properties of polymers)

Nomenclature

w	Width of the IPMC sheet (m)
d	Thickness of the IPMC sheet (m)
l	Length of free part of the IPMC sheet (m)
l_c	Length of the fixed part of the IPMC sheet (m)
l_F	Length of free IPMC section, which is also loaded (m)
s	Natural parameter of the curve representing IPMC sheet (m)
s_F	Position on the sheet, where force is applied (m)
B	Bending stiffness ($N \cdot m^2$)
E	Equivalent Young modulus (Pa)
F_{sheet}	Force applied to the sheet (N)
F	Force applied to the object (N)
ϕ	Angle between the normal of the sheet and the tangent of the trajectory (rad)
R	Radius of the trajectory (m)
p	Position of the object on the trajectory (m)

$k_0(s)$	Initial curvature (m^{-1})
$\hat{\alpha}_0$	Initial angular deflection in the position of the object (rad)
$k(s)$	Curvature (m^{-1})
$\alpha(s)$	Tangential angle (rad)
$\hat{\alpha}$	Angular deflection in the position of the object (rad)
$\vec{P}(s)$	Position vector (m)
$\bar{\vec{P}}$	Position vector of the center of the loaded section of IPMC (m)
$M(s)$	Bending moment caused by force applied to the sheet ($N \cdot m$)
$\bar{M}$	Mean bending moment of the loaded section of IPMC caused by force applied to the sheet ($N \cdot m$)
$M_e(s)$	Electrically induced bending moment (EIBM) (input voltage is unspecified) ($N \cdot m$)
$M_e^+(s)$	EIBM at +2V ($N \cdot m$)
$M_e^-(s)$	EIBM at −2V ($N \cdot m$)
M_e	Constant EIBM (input voltage is unspecified) ($N \cdot m$)
M_e^+	Constant EIBM at +2V ($N \cdot m$)
M_e^-	Constant EIBM at −2V ($N \cdot m$)
$\vec{T}(\theta)$	Tangent vector, where θ is tangential angle (unitless)
$\vec{N}(\theta)$	Normal vector, where θ is tangential angle (unitless)

1 Introduction

IPMC (Ionomeric polymer metal composite) is a type of an electroactive material that bends in electric field [1, 2]. It consists of a thin swollen polymer film, such as Nafion™, filled with water or ionic liquid. Both sides of the polymer film are plated with thin metal electrodes. Voltage applied between the surface electrodes causes migration of ions inside the structure of the polymer, which in turn causes the mechanical bending of the sheet as shown in figure 1. The direction of bending depends on the polarity of the applied electric field.

(a) (b) (c)

Figure 1 IPMC with (a and c) and without (b) electrical stimulation.

This paper investigates the mechanical model of an IPMC sheet in the cantilever beam configuration. It is a physical model, with electrically induced bending moment (from now on abbreviation EIBM is used) as an input parameter and enables to calculate the shape of the sheet.

Various mechanical and electromechanical models of IPMCs have been developed by a number of authors, emphasizing different features [3-11]. Table 1 summarises the features usually considered in those models. Table 2 represents the comparison between our paper and the previously described models of IPMC. Besides the features depicted in the table the models can differ by other details. For example, [3] describes the dependency from time of elasticity and EIBM, model in [6] is two-dimensional. While in all the other papers the load is crosswise with the beam, in [11] axial load is considered.

The main motivation of our work is to develop a mechanical model of the IPMC actuator that as precisely as possible describes the quasi-static behaviour of IPMC cantilever beam. For example, we considered it to be important to describe the behaviour at large deflections and with varying load positions. It makes it possible to use the model for a great variety of applications. Also, we are interested in applying the model in case of materials with various properties. For example, it is often the case that the model has an initial bending curvature or the EIBM varies along the surface but only few models consider these assumptions.

Another feature of our model is that it permits modeling an IPMC actuator with a rigid passive elongation. The reason for doing this is that we want to investigate how the elongation changes the behaviour of the actuator. In this paper we show that the elongation makes the behaviour of the actuator more linear and therefore easier to control.

Several models [3,4,7,8] assume the linear behaviour of the IPMC actuator. In our work we do not make such an assumption as it is rather restrictive. For example, it permits modeling the IPMC only at small deflections but we are also interested in cases where the bending is large and non-uniform. Assuming non-linear behaviour makes it possible to investigate a more general case. Furthermore, it makes it also possible to determine to which extent the behaviour of the IPMC can be linearized. In our paper we show that this is the case if, for example, a short sheet with a rigid elongation is considered.

The model presented in this paper permits comparing actuators with various properties. This would facilitate general understanding how does the behaviour change when the properties changes and thus help to design better actuators. In this paper we represent the comparison of actuators with and without elongation. We allow EIBM to wary along sheet and we also investigate the possibility that EIBM is constant.

The limitation of our model is that it does not consider viscoleasticity and other dynamic properties of IPMC cantilever beams. To elaborate the model so that it also accurately describes the dynamic behaviour of the sheet is a direction of our future work.

The remainder of the paper is organized as follows. In section 1 the system we are modeling is decribed. Experiments made are discussed in section 3. In section 4 the model is validated against experimental data. In section 5 possible applications and future work is discussed. Finally, concluding remarks are provided in section 6.

Table 1 Features.

	Feature	Description
1	Non uniform bending	Is the curvature of the IPMC sheet assumed to be constant or not? In general only free bending curvature may be considered approximately constant. And even then only if EIBM is constant and the IPMC sheet is moving slowly. As soon as the IPMC sheet is loaded, the curvature will vary along the sheet.
2	Initial Curvature	IPMC sheets may be initially curved even when neither force nor electric current is applied (see table 3). Alternately the model may be suited to describe only nearly perfectly straight IPMC sheets.
3	Big deformations	In this paper all deformations, that result deflection angles bigger than 90 degrees are considered big. Models, which do not support big deformations, have gradually increasing modeling error as deflection angle approaches 90 degrees.

4	Non uniform EIBM	Several authors report, that EIBM varies along the sheet [12-14]. Alternately in case of small pieces of IPMC, small currents and good electrode layers the EIBM can be considered uniform.
5	Force output	IPMC actuator can be used to apply force an object (load cell). Alternately only free bending of the IPMC sheet may be modeled.
6	Varying Load position	When doing work IPMC actuator pushes an object along a trajectory. So the IPMC sheet applies force to the object in different positions. Alternately the force may be determined only on the same plane with the contacts.
7	Elongation	May some rigid elongation be attached on top of the IPMC sheet. The usefulness of that kind of construction is discussed in this paper.
8	Linearity	Can the model be defined by linear differential equations? Linearity is very important if we want to control the IPMC sheet in real time.
9	Dynamic behavior	Does the model describe the movements of the IPMC sheet? Alternatively the model may only describe the static equilibrium of IPMC sheet. That kind of model can be used to describe quasi-static movements of the sheet – movements that are so slow, that inertial and drag forces can be neglected.

Table 2 Comparison of models.

	Feature \ Paper	this	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
1	Non uniform bending	✓	✓	✓	✓	✓	✓		✓		✓
2	Initial Curvature	✓								✓	
3	Big deformations	✓			✓				✓	✓	
4	Non uniform EIBM	✓							✓		
5	Force output	✓	✓	✓	✓	✓	✓	✓			
6	Varying Load position	✓			✓	✓					
7	Elongation	✓						✓			
8	Linearity		✓	✓			✓	✓			
9	Dynamic behavior			✓			✓		✓		✓

2 The model

This section represents the mechanical model of the IPMC cantilever actuator with the properties given above.

Our objective is to model a situation where an IPMC sheet in a cantilever configuration manipulates an object. The sheet can have an absolutely rigid elongation attached to the tip. In this paper we study quasi-static movements of the object. Thus, we model the sheet in a static equilibrium state.

A part of the IPMC sheet is fixed between contacts at clamps or the elongation. Mechanics of that part does not need to be modeled. The rest of the sheet can bend and is called the free part of the IPMC sheet. The length of the free part is l. Formally every IPMC sheet in this model has an elongation with an infinite length. It helps to model elongated sheets but can equally be used for modeling sheets without an elongation.

The shape of the IPMC sheet is modeled as a curve on a plane (see figure 2.a). The curve (or the neutral curve) is a projection of the neutral surface of the sheet – the surface that does neither contract nor expand. The neutral surface is assumed to be cylindrical. The only allowed deformation is bending. The neutral curve is extended by a ray (see figure 2.a) which represents the elongation.

Throughout this paper s denotes the natural parameter of the neutral curve that specifies a position on the sheet at a distance s from contacts along the curve. Neutral curve is defined with its curvature

$k(s)$ (see figure 2.b). The curvature of the elongation $s > l$ is zero. The curvature of the IPMC part of the curve is given by:

$$\forall s \le l: \quad k(s) = k_0(s) + \frac{M_e(s) + M(s)}{B} \tag{1}$$

where $k_0(s)$ is the initial curvature of the IPMC sheet, $M(s)$ is bending stiffness and $M_e(s)$ is electrically induced bending moment (EIBM). $M(s)$ is bending moment caused by force applied to the sheet. $M(s)$ is equal to the area of the parallelogram in figure 2.b.

Force applied to the sheet is F_{sheet} (see figure 2.b). Then force applied to the object is $-F_{\text{sheet}}$. The object needs not to be specified. The only thing we need to know is that the point of contact is located on a circular trajectory in a position p (see figure 2.b). The center of the circle is at the contacts of the sheet and the radius is R. The contact point on the neutral curve is s_F (see figure 2.a).

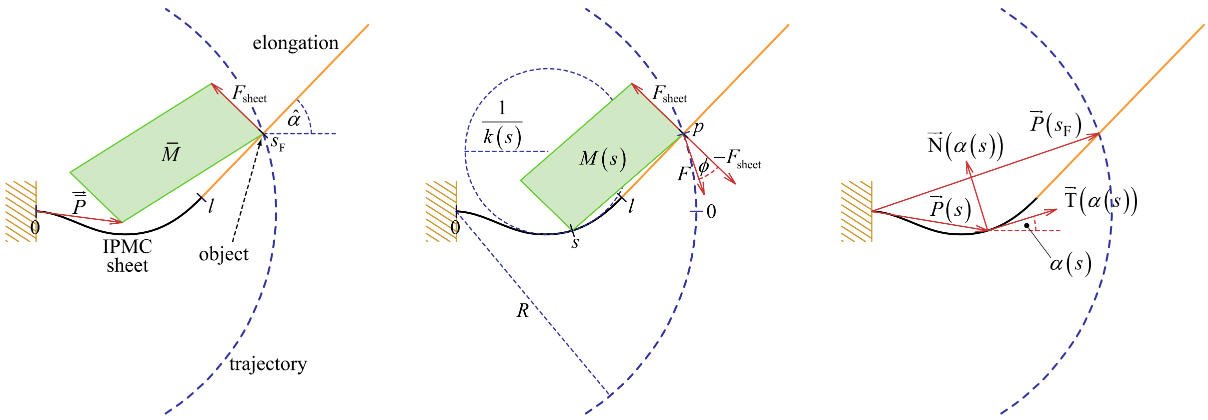


Figure 2 Geometric definitions of parameters

The model can be used for calculating the deflection angle $\hat{\alpha}$ at the position of the object. In general there is no nice analytical formula for that. Please refer [15] for the corresponding numerical algorithm. However if we may assume that EIBM is constant ($M_e(s) = M_e$) it holds that:

$$\hat{\alpha} = \hat{\alpha}_0 + \frac{l_F \cdot (M_e + \bar{M})}{B}, \tag{2}$$

where

$$l_F = \min(l, s_F), \tag{3}$$

$\hat{\alpha}_0$ is the initial angular deflection in the position of the object and $\bar{M}$ is mean bending moment (see figure 2.a). Please refer to the appendix B for derivation of (2).

The mean bending moment $\bar{M}$ caused by external force equals to the bending moment in the center of the bendable section - $\bar{\bar{P}}$ (see figure 2.a). $\bar{M}$ can be calculated with

$$\bar{M} = \left\| \left(\vec{P}(s_F) - \bar{\bar{P}}\right) \times \left(F_{\text{sheet}} \cdot \vec{N}(\hat{\alpha})\right) \right\|, \tag{4}$$

where $\vec{P}(s_F)$ points to the location of the object (see figure 2.c) and $F_{\text{sheet}} \cdot \vec{N}(\hat{\alpha})$ is force vector applied to the sheet. Please refer to the appendix C for derivation of (4).

Equations (1) and (2) are the basic equations of the model. Please refer Appendix A for the complete list of equations which define the model.

Our further objective is to calculate the quasi-static work done by moving an object along the arc. The sheet is assumed to be in a frictionless contact with the object thus reaction $-F_{\text{sheet}}$ is perpendicular to the sheet. To calculate the output work, we need to find force component F (see figure 2.b) that is in direction of the tangent of the arc. For the rest of the paper, if we talk about force we mean F. The force is given by

$$F = -F_{\text{sheet}} \cdot \cos(\phi), \tag{5}$$

where ϕ is angle between the forces $-F_{\text{sheet}}$ and F (see figure 2.b).

If we double the width of the IPMC sheet, it is equivalent to two sheets working in parallel. Force needed to bend would double. The same holds for stiffness and EIBM. Output force F, bending stiffness B and EIBM $M_{\text{e}}(s)$ are proportional with the width w of the sheet. B and $M_{\text{e}}(s)$ normalized to the width of the sheet characterize the properties of the IPMC material.

IPMC is a sandwich with cracked surface and several layers with different elastic modulus. The Young modulus of a homogeneous sheet with same dimensions and similar stiffness can be calculated using the equation

$$E = \frac{12 \cdot B}{w \cdot d^3}, \tag{6}$$

where d is thickness of the IPMC sheet. For non homogeneous material like IPMC this parameter is called effective Young modulus or equivalent Young modulus. It can be used to characterize the properties of the IPMC material.

3 Experiments

This section describes the experiments made to verify the model introduced in the previous section and to compare actuators with and without elongation. First experimental setup is discussed. Then experimental data is introduced.

3.1 Experimental setup

The system setup allows us to apply voltage to an IPMC sheet, with or without elongation, and measure force and neutral curve in arbitrary position of the object (in our case the load cell).

Our system consists of six basic parts (see figure 3):

1. A sheet either consisting entirely of IPMC material or a combination of IPMC material and a plastic elongation attached to it with bolt and nut.
2. A clamp which holds the sheet. The jaws of the clamp are made of gold and serve as an electrical contact. It also holds the sheet so that it bends in the horizontal plane. The horizontal motion assures that gravity does not affect the experimental results.
3. A Transducer Techniques GSO-10 load cell is used to measure the force applied by the sheet. Maximum force that can be measured is about 0.1N. The sheet is held between two rolls, to achieve a frictionless contact. Between the experiments the load cell can be repositioned with respect to the clamp.
4. Black&white camera Dragonfly EXPRESS is used to record the sheet. The resolution of the camera is 640×480 pixels and frame rate used is 25 fps. From acquired images the neutral curve can be determined.
5. The inputs and outputs of the system are controlled and monitored with a PC running LabView 7. The input voltage of the sheet is controlled with National Instruments I/O board PCI-6703. Load cell is attached to DAQ board PCI-6034. The camera Dragonfly EXPRESS is connected to the IEEE 1394b port.
6. The control signal generated by I/O board is amplified with the amplifier LM675 (National Semiconductor).

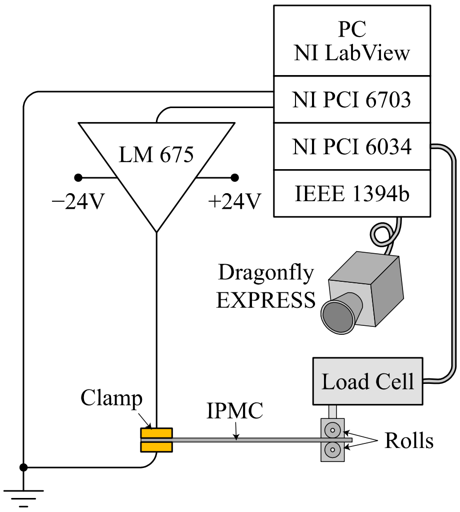
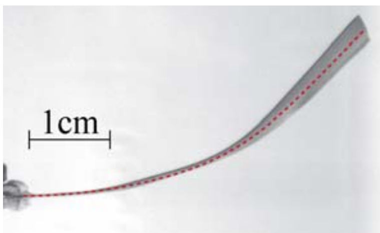
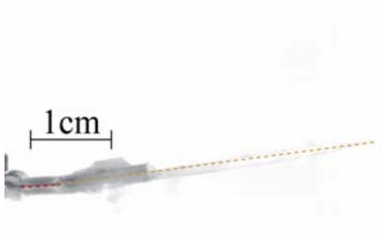


Figure 3 System setup.

Ready-made Pt-coated Na-ions containing MuscleSheet™ IPMCs, purchased from BioMimetics Inc, where used for the experiments.

Table 3 summarises information about the two sheet configurations used in experiments. Long IPMC sheet is initially twisted but will be straight when held between rolls. The short IPMC sheet is cut from the first part of the long IPMC sheet after conducting the experiments with the long one.

Table 3 Sheet configurations.

Configuration	Long IPMC sheet	Short IPMC sheet with the plastic elongation
Side view of the sheet and the initial neutral curve (the red line) with curvature - $k_0(s)$.		
Length of the freely bending IPMC part - l	50 mm	4.5 mm
Length of the part of the IPMC sheet that is fixed - l_c	1.5 mm	3.5 mm
Width of the IPMC sheet - w	11 mm	11 mm
Thickness of the IPMC sheet - d	0.21 mm	0.21 mm

Parameter l_c specifies the length of the IPMC sheet that is fixed between contacts at clamps or elongation. This parameter does not influence the behavior of the sheet. Parameters w and d , on the other hand, have a direct impact to the sheets behavior. They are not explicitly used in the model but model parameters B and $M_e(s)$ are related to them. They are measured to assess the parameters of the IPMC material.

Throughout this paper we call actuators corresponding to sheets “long sheet” and “short sheet” respectively. Throughout this paper the radius of the trajectory is $R = 40\,\text{mm}$. In experiments force is measured in 7 positions located on the trajectory (see figure 4).

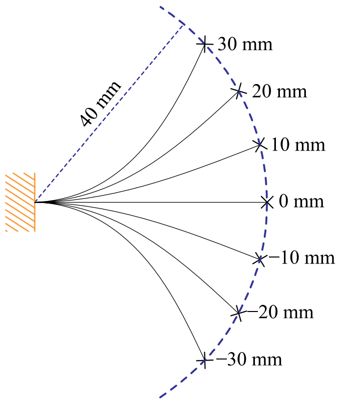


Figure 4 Positions on trajectory, where force and shape of the sheet are measured. Measure denotes distance along the trajectory form position zero.

For each of the:

- two sheet configurations,
- 7 trajectory positions and
- 3 driving voltages (+2V, 0V and −2V)

force F and the neutral curve of the sheet are measured. From the neutral curve, the curvature $k(s)$ can be extracted.

3.2 Implementation details

For every sheet configuration and position of the trajectory the force and the neutral curve at 3 driving voltages (+2V, 0V and −2V) is measured. Voltage input and timing of important readings can be seen in figure 5. Corresponding current output can be seen on figure 6. First 10 readings are taken at 0V. After these 10 readings, the readings are taken alternatively at +2V and -2V. Force and neutral curve are measured 0.5s after the start of the impulse. To enable signal filtering the total length of the input pulse is 0.6s.

Electric current at 2V dehydrates the IPMC sheet [1, 16]. Changes in the hydration level of the IPMC sheet cause changes in bending stiffness and response to electrical stimulation. Therefore the experiments where kept as short as possible and the IPMC sheet was moistened between the measurements.

After every pulse water is unevenly distributed within the material. The first 2 pulses are neglected because initial conditions are different compared to those of the following pulses.

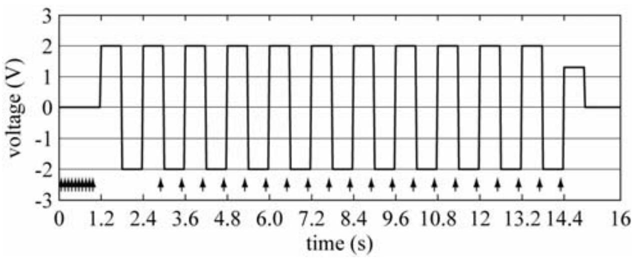


Figure 5 Input voltage and times of moments of important readings.

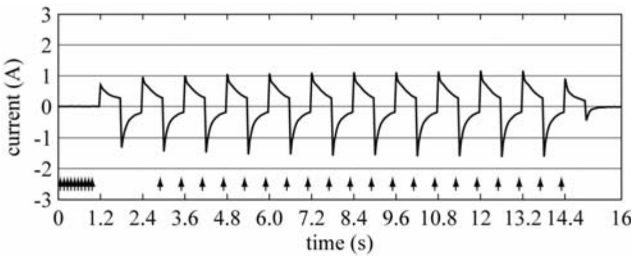


Figure 6 Output current in case of the long IPMC sheet.

We measure force component in direction of the tangent of the trajectory. The load cell measures the component of the force that is perpendicular to it and has to be positioned perpendicular to the tangent of the trajectory. Ten readings are taken and the mean value is used as the result.

Ten images are saved synchronously with force measurements. Mean of these images as light intensity matrixes is computed. This operation may cause motion blur. Neutral curve of the IPMC part of the sheet is interpolated with minimum variation curvature curve [17]. Curvature of such a curve can be easily analytically revealed.

3.3 Experimental data

Figure 7 represents the neutral curves measured. Measured forces are presented on figure 9. Please refer to [18,19] for the videos of the experiments of long and short sheet respectively in position of the trajectory $p = 0$.

On the videos [18,19] and in figure 7 can be seen, that with different voltages the static equilibrium state of the sheet differs. In case of a long sheet the difference is big. In case of a short sheet with elongation the difference is barley notable. When considering moving sheets the shape of the sheet is an input parameter that we need to know in order to calculate the output force. In case of short sheet with elongation the shape of the sheet does not vary so much and can be easily estimated. In case of the long sheet estimation of the shape of the sheet is more difficult.

Readings taken in position 30 mm of the trajectory are invalid because the sheet systematically got stuck between the rolls.

Readings taken in position 30 mm of the trajectory are invalid because the sheet systematically got stuck between the rolls.

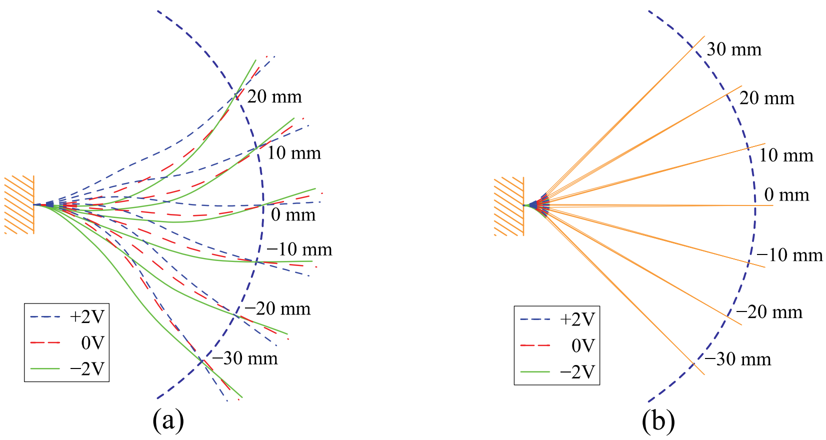


Figure 7 Neutral curves of long sheet (a) and short sheet (b).

4 Model validation

In this section we validate the model of the IPMC actuator by extracting the parameters for the model and comparing the simulation result with the experimental results of the previous section.

4.1 Parameter identification

The parameters that have to be extracted from the experiments for both of the sheet configurations are bending stiffness and the electrically induced bending moment (EIBM). Each of three driving voltage, used in the previous section to drive the actuator, cause a different EIBM, the bending stiffness however is always the same.

At 0V $M_\mathrm{e} = 0$ (no EIBM). The following equation derived from (2) can be used to calculate bending stiffness:

$$B = \frac{l_\mathrm{F} \cdot \bar{M}}{\hat{\alpha} - \hat{\alpha}_0}. \tag{7}$$

There are number of measurements (corresponding to different positions on trajectory) that each give sample of bending stiffness. The weighted average is used as the result. A weight coefficient $\bar{M}$ is used so that the experiments with a higher mean bending moment would contribute more to the result.

Varying electrically induced bending moment in case of both electrical stimulations can be calculated using equation derived form (1):

$$M_\mathrm{e}(s) = B \cdot \left(k(s) - k_0(s)\right) - M(s). \tag{8}$$

For calculating the constant electrically induced bending moment the equation derived form (2) can be used:

$$M_\mathrm{e} = \frac{B \cdot \left(\hat{\alpha} - \hat{\alpha}_0\right)}{l_\mathrm{F}} - \bar{M}. \tag{9}$$

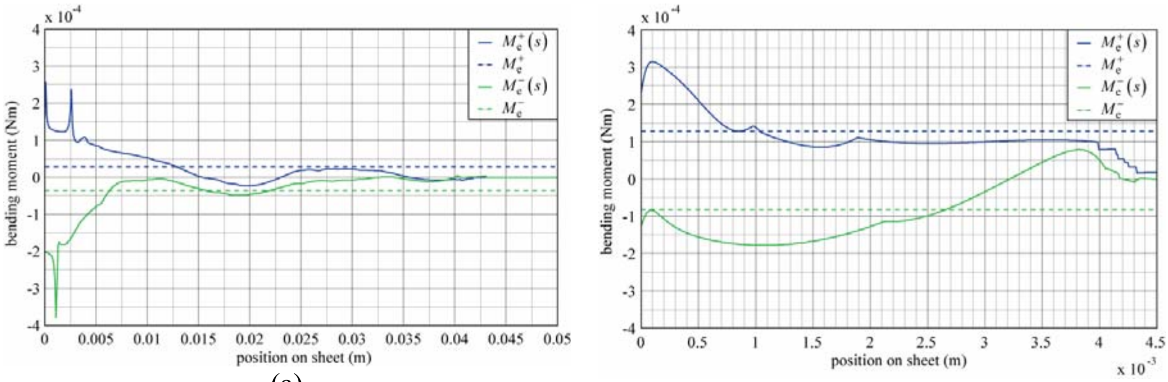
In both cases (varying and constant EIBM) there are again number of measurements that each give sample of EIBM. Arithmetical average is used as the result.

4.2 Identified parameters of the model

Extracted parameters are presented in table 4 and in figure 8.

Table 4 Extracted parameters.

Configuration	Notation (equation)	Long IPMC sheet	Short IPMC sheet
Bending stiffness	B	$2.03 \cdot 10^{-6}$ N·m^2	$1.21 \cdot 10^{-6}$ N·m^2
Bending stiffness normalized to the width of the sheet	$\frac{B}{w}$	$1.84 \cdot 10^{-4}$ N·m	$1.10 \cdot 10^{-4}$ N·m
Equivalent Young modulus	E	236 MPa	147 MPa
Constant EIBM at +2V	M_e^+	0.029 mN·m	0.127 mN·m
Constant EIBM at −2V	M_e^-	–0.036 mN·m	–0.082 mN·m
The mean absolute value of constant EIBM	$\frac{M_\mathrm{e}^+ - M_\mathrm{e}^-}{2}$	0.032 mN·m	0.104 mN·m
The mean absolute value of constant EIBM normalized to the width of the sheet	$\frac{M_\mathrm{e}^+ - M_\mathrm{e}^-}{2 \cdot w}$	2.95 mN	9.47 mN



(a) (b)

Figure 8 Electrically induced bending moment of long sheet (a) and short sheet (b).

Observations

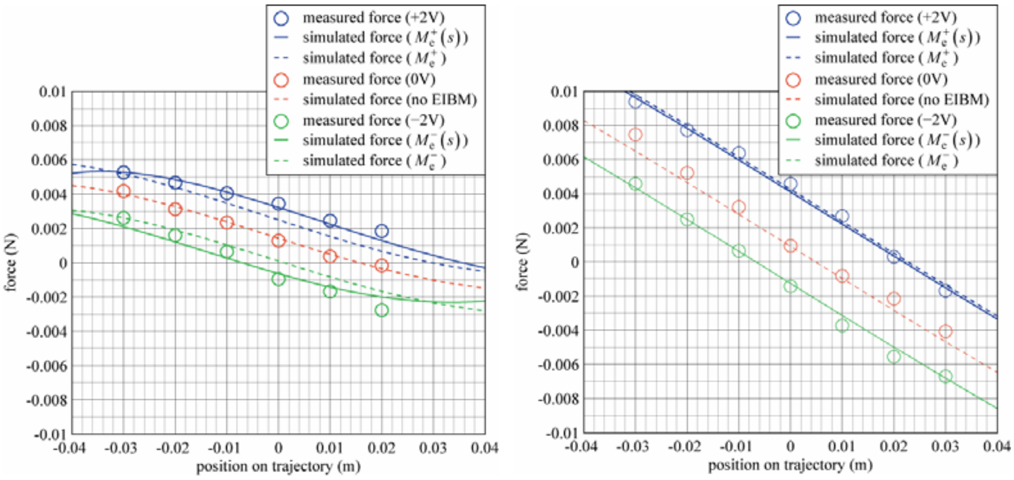
1) Long sheet appears to be stiffer then short sheet (although the short IPMC sheet is cut from the first part of the long IPMC sheet). It can be explained by changes in hydration level [1, 16]. Short IPMC sheet is surrounded by water while long sheet is not (for side view of the sheet see table 3). Short IPMC sheet is surrounded by water because elongation is near to the contacts and because surface tension of water.
2) As s increases the varying EIBM tend to converge to zero. It can be explained by the fact that moving toward the top of the sheet potential between surfaces drop because of the surface resistance [12-14]. The smaller potential, the smaller EIBM.
3) The mean absolute value of a constant EIBM is smaller for the long sheet. The top part of the long sheet has small EIBM and thereby the makes sheet weaker as the whole.
4) EIBM (both varying and constant) at voltages +2V and –2V do not have the same absolute value. This reflects the asymmetry of the material.
5) Varying EIBM changes polarity or has peaks at some positions. This is caused by errors in curvature measurements.

4.3 Simulations

With the parameters extracted in the previous section and the parameters from table 3 the simulations are conducted and the simulation results are validated against the experimental data.

The system of equations given in appendix A. can be solved using customized numerical integration technique. The challenge in our case is that, at first we know only on point of the function we are integrating. The rest of the points are found in course of the integration. The algorithm is implemented in LabView 8.0. Please refer to [15] for documentation and source code.

In figure 9 real and simulated output forces at different electrical stimulations can be seen. In table 5 real and simulated neutral curves at +2V in position 0mm of the trajectory can be seen. Root mean squared error of the neutral curve for the IPMC part of the sheet is also given. Table 6 gives root mean squared errors of the neutral curve and of the output force over all positions of trajectory.



(a) (b)

Figure 9 Real and simulated forces of long sheet (a) and short sheet (b).

Table 5 Real and simulated neutral curves and root mean squared errors at +2V in position 0m of the trajectory.

Configuration	Long IPMC sheet	Short IPMC sheet with plastic elongation
Actual	 RMSE = 0 mm	 RMSE = 0 mm
With varying EIBM.	 RMSE = 0.44 mm	 RMSE = 0.05 mm
With constant EIBM.	 RMSE = 2.23 mm	 RMSE = 0.09 mm

Table 6 Root mean squared errors.

Configuration		Long IPMC sheet	Short IPMC sheet with plastic elongation
Varying EIBM	Neutral curve	0.81 mm	0.13 mm
	Force	0.322 mN	0.424 mN
Constant EIBM	Neutral curve	2.13 mm	0.18 mm
	Force	0.622 mN	0.422 mN

Varying EIBM-s ($M_e^+(s)$ and $M_e^-(s)$) are calculated with (6) based on curvature. Constant EIBM-s (M_e^+ and M_e^-) are calculated with (7) based on deflection angle. It is easier to measure the angle than the curvature but comparison of the experiments and simulations indicates that in case of the long sheet the use of constant EIBM causes larger modeling errors (see table 6). In case of the short sheet, the use of constant EIBM gives as good results as the use of varying EIBM does.

Position-force relation is linear in case of the short sheet. This is important with respect to precision control of such an actuator. As controlling linear time invariant system (LTI) is very easy, stabilizing nonlinear system (like actuator with long sheet) can be very difficult.

The initial position of the object (without electrical stimulation) is 15mm in case of the long sheet and 5mm in case of the short sheet (see figure 9). The short sheet can apply more force in the initial position, than long sheet is able to. The main reasons are:

a) most of the of the work is done by IPMC material near the input contacts, where the bending moment caused by the external force is the largest and
b) EIBM tends to converge to zero in direction away from the contacts [12-14].

5 Possible applications and future work

The model presented and validated in this paper can be used to determine the properties of the actuator and to optimize the construction of the actuator.

To determine the properties of the actuator we need to know l, $k_0(s)$, R and R. For every position p of the trajectory and for every EIBM $M_e(s)$ the model determines the shape of the sheet $k(s)$ and the force F applied to the object. Please refer to [15] for the corresponding algorithm.

Usually we are only interested in a specific section of positions of the trajectory, the working section and a set of EIBM-s determined by the material. Given the working section and a set of realistic EIBM-s the model enables us to do the following:

1. To find the linear fit of the force F as a function of p in the given working section. If the mean squared error is small enough the linear fit can be used for creating a linear controller.
2. To find the maximal difference of the shapes of the sheet at extreme opposite realistic EIBM-s. The less the shape of the sheet varies the easier it is to control it.
3. To find the minimal length of the sheet, capable to reach the object in every position of the working section.
4. To find the maximal force that can be applied in the direction of the tangent and the normal of the trajectory.
5. To find the maximal magnitude of the force that can be applied in both directions within the working section.

Further on, the model also enables us to find the minimal area of an IPMC sheet, capable of applying a predefined force within a predefined working section. As the force per area of the sheet does not depend on the width of the sheet, the required parameter is the optimal length of IPMC sheet. After the optimal length is found, the width of the sheet can be chosen according to the required output force. To investigate how and when the elongation as a construction element improves the performance of the actuator and the optimal configuration of the actuator is the topic of our active research.

6 Conclusions

We presented a model of a cantilever IPMC actuator with a varying electrically induced bending moment. We validate the model against two actuators: a long IPMC sheet and a short IPMC sheet with a rigid elongation. The experimental results show that for the short sheet the position-force relationship is linear whereas the position-force relationship of the long sheet is non-linear.
This paper presents a static analysis of IPMC mechanics. It enables to compute free bending curvature and force applied to an object in a static equilibrium state. A dynamic model which considers motion, masses and viscoelasticity, is a subject of future study. However, for cases where accelerations and velocities are small enough, the static model is shown do be accurate. It is also suitable for finding the optimal design of the actuator.

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Appendix A. Formal definition of the model

The relations between parameters are defined by following equations:

$$\vec{\mathrm{T}}(\theta)=\begin{pmatrix}\cos(\theta) & \sin(\theta) & 0\end{pmatrix}, \tag{10}$$

$$\vec{\mathrm{N}}(\theta)=\begin{pmatrix}-\sin(\theta) & \cos(\theta) & 0\end{pmatrix}, \tag{11}$$

$$l_{\mathrm{F}}=\min(l,s_{\mathrm{F}}), \tag{12}$$

$$F=-F_{\mathrm{sheet}}\cdot\cos(\phi), \tag{13}$$

$$\phi=\hat{\alpha}-\frac{p}{R}, \tag{14}$$

$$\vec{P}(s_{\mathrm{F}})=R\cdot\vec{\mathrm{T}}\left(\frac{p}{R}\right), \tag{15}$$

$$k(s)=\begin{cases}k_0(s)+\dfrac{M_{\mathrm{e}}(s)+M(s)}{B} & s\le l\\ 0 & s>l\end{cases}, \tag{16}$$

$$\alpha(s)=\int_0^s k(u)\,du, \tag{17}$$

$$\hat{\alpha}=\alpha(s_{\mathrm{F}}), \tag{18}$$

$$\hat{\alpha}_0=\int_0^{l_{\mathrm{F}}} k_0(u)\,du, \tag{19}$$

$$\vec{P}(s)=\int_0^s \vec{\mathrm{T}}(\alpha(u))\,du, \tag{20}$$

$$\overline{\vec{P}}=\frac{1}{l_{\mathrm{F}}}\cdot\int_0^{l_{\mathrm{F}}}\vec{P}(u)\,du, \tag{21}$$

$$M(s)=\begin{cases}\left\|\left(\vec{P}(s_{\mathrm{F}})-\vec{P}(s)\right)\times\left(F_{\mathrm{sheet}}\cdot\vec{\mathrm{N}}(\alpha(s_{\mathrm{F}}))\right)\right\| & s\le s_{\mathrm{F}}\\ 0 & s>s_{\mathrm{F}}\end{cases}, \tag{22}$$

$$\overline{M}=\frac{1}{l_{\mathrm{F}}}\cdot\int_0^{l_{\mathrm{F}}}M(u)\,du. \tag{23}$$

For realistic results $l>0$, $B>0$, $R>0$ and $-\frac{\pi}{2}<\phi<\frac{\pi}{2}$. To be able to use cross product, all the vectors, except the result of the cross product, are 3-dimensional with zero z coordinate.

Appendix B. Derivation of equation (2)

Assuming $M_{\mathrm{e}}(s)=M_{\mathrm{e}}$ from (1) we get

$$\forall s\le l_{\mathrm{F}}\quad k(s)=k_0(s)+\frac{M_{\mathrm{e}}+M(s)}{B}\quad\Rightarrow$$

$$\int_0^{l_{\mathrm{F}}}k(u)\cdot du=\int_0^{l_{\mathrm{F}}}\left(k_0(u)+\frac{M_{\mathrm{e}}+M(s)}{B}\right)du\quad\Leftrightarrow$$

$$\int_0^{l_F} k(u)\cdot du = \int_0^{l_F} k_0(u)\cdot du + \frac{l_F \cdot M_e + \int_0^{l_F} M(u)\cdot du}{B} \quad \Leftrightarrow$$

$$\alpha(l_F) = \hat{\alpha}_0 + \frac{l_F \cdot \left(M_e + \frac{1}{l_F}\cdot \int_0^{l_F} M(u)\cdot du \right)}{B} \quad \Leftrightarrow$$

$$\alpha(s_F) = \hat{\alpha}_0 + \frac{l_F \cdot \left(M_e + \bar{M} \right)}{B} \quad \Leftrightarrow$$

$$\hat{\alpha} = \hat{\alpha}_0 + \frac{l_F \cdot \left(M_e + \bar{M} \right)}{B},$$

which is (2). In the above first we integrate both sides of the equation. On the second step we use linearity property of integration. On the third step we use (17) and (19). On the fourth step we use (16), (17) and (23). On the fifth step we use (18).

Appendix C. Derivation of equation (4)

From (23) we get

$$\bar{M} = \frac{1}{l_F}\cdot \int_0^{l_F} M(u)\, du$$

$$= \frac{1}{l_F}\cdot \int_0^{l_F} \left\| \left(\vec{P}(s_F) - \vec{P}(u) \right) \times \left(F_{\text{sheet}} \cdot \vec{\mathrm{N}}\left(\alpha(s_F) \right) \right) \right\| du$$

$$= \left\| \left(\vec{P}(s_F) - \frac{1}{l_F}\cdot \int_0^{l_F} \vec{P}(u)\cdot du \right) \times \left(F_{\text{sheet}} \cdot \vec{\mathrm{N}}(\hat{\alpha}) \right) \right\|$$

$$= \left\| \left(\vec{P}(s_F) - \bar{\vec{P}} \right) \times \left(F_{\text{sheet}} \cdot \vec{\mathrm{N}}(\hat{\alpha}) \right) \right\|,$$

which is (4). In the above the first equality is from (12) and (22), the second equality follows from linearity of cross product and integration and the third equality is from (21).

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