Physics 740: Spring 2006: 01/31/07

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Transport Coefficients: a Review. Transport coefficients arise in the description of currents, charge current, energy current, number current, momentum current. Systems carrying currents are open, something goes in/comes out, and out of equilibrium. Systems carrying a steady current, the usual context for measurement of transport coefficients, are said to be in a non-equilibrium steady state (NESS). The physics context for careful calculation of transport coefficients is non-equilibrium statistical mechanics/irreversible thermodynamics. (Onsager's Nobel prize, in chemistry, is for his work on this subject and not for his exact solution to the d=2 Ising model.)

We are going to do some back of the envelope calculations to see the basics of what is involved. A few careful calculations will follow in a few days using kinetic theory.

Electrical conductivity. Keep the electrical conductivity in mind as an example. To measure σ in $J = \sigma E$ you think $V = RI$ and make the measurement shown in the figure below. As you know a charge current carrying system generates heat, the electric field is doing work, V^2/R, so to make a proper measurement you control the temperature of the system to find $R(T)$ or $\sigma(T)$. [One way to make a furnace is to drive a large current through a resistor that you surround with a thermal insulator so that it must heat up.]

Mean free path. As background for the back of the envelope calculations we need the concept of mean free path, the distance a particle travels before undergoing collision. In a not too dense gas the calculation is as sketched in the figure here. A particle is taken to sweep out a cylindrical volume $\sigma v t$ in time t; the endcap area is equal to the particle-particle cross-section, σ, and v is the average velocity of a particle. The density of particles in the gas is $n = N/V$ so the particle has $\mathcal{N} = n\sigma v t$ collisions in time t or collisions at rate $w = \mathcal{N}/t$ sec^{-1}. The inverse of w is the time between collisions, $\tau = w^{-1}$. The typical distance traveled between collisions is the mean free path $l = v\tau$,

$$l = \frac{1}{n\sigma}. \tag{1}$$

TABLE I: Transport Coefficients.

force	current	defining equation
$E = -\nabla V$	J_Q	$J_Q = \sigma E$
∇n	J_n	$J_n = -D\nabla n$
∇T	Q	$Q = -\kappa \nabla T$
$\partial u_y/\partial x$	Π_{yx}	$\Pi_{yx} = -\eta \partial u_y/\partial x$

Examples. Here are 4 examples of transport calculations.

In each case a current is the response to a driving force. Sometimes the driving force is obvious, as in the case of an electric field. Usually it is more subtle, e.g., the temperature gradient is the driving force for a heat current, etc. The quantity which relates the current to the driving force is a transport coefficient.

1. σ: the electric field creates a force which works on a particle for time τ producing the change in velocity $\Delta v = QE\tau/m$ (impulse-momentum). Thus $v_L = v_R + \Delta v$ and
$$J_Q \approx n\frac{Q^2\tau}{m}E \rightarrow \sigma \approx n\frac{Q^2\tau}{m}. \tag{2}$$

2. D_n: the density on the left/right is $n \mp \delta n$, $\delta n = (\partial n/\partial x)l$, where l is the mean free path. Then,
$$J_n \approx -vl\frac{\partial n}{\partial x} \rightarrow D_n \approx vl. \tag{3}$$

3. κ: the temperature on the left/right is $T \mp \delta T$, $\delta T = (\partial T/\partial x)l$, where l is the mean free path. Then,
$$Q \approx -nk_Bvl\frac{\partial T}{\partial x} \rightarrow \kappa \approx nk_Bvl. \tag{4}$$

4. η: u_y on the left/right is $u_y \mp \delta u_y$, $\delta u_y = (\partial u_y/\partial x)l$, where l is the mean free path. Then,
$$Q \approx -nmvl\frac{\partial u_y}{\partial x} \rightarrow \eta \approx nmvl. \tag{5}$$