Physics 740: Spring 2006:

Afternote.4.tex 03/05/07

Lift: the Bernoulli argument.

1. Λ. The dimensionless measure of the influence of the circulation on the lift of the *Bernoulli cylinder* is Λ. [*Bernoulli cylinder, Bernoulli lift?* ↔ cylinder lifted by forces found from an argument that uses the Bernoulli equation.] Look at the definition

$$\Lambda = \frac{\Gamma}{2\pi v_\infty}. \qquad (1)$$

In the frame moving with the cylinder the air is seen rushing past. In class we had the air moving from left to right. So in the frame of the cylinder the air is seen to pass over (under) from left to right. It has velocity v_∞ at the top and bottom of the cylinder for $\Lambda = 0$. Formally you subtract $\mathbf{v}_\infty = v_\infty \mathbf{e}_x = v_\infty \cos\theta \ \mathbf{e}_r - v_\infty \sin\theta \ \mathbf{e}_\theta$ from Eqs. (41) and (42) in **P740.10.tex**.

To increase the velocity of the air above the cylinder you rotate the air in the clockwise direction. From Eq. (21) and Eq. (36) in **P740.10.tex** $\Gamma/(2\pi)$ is the tangential velocity of the rotated air at the cylinder radius. That is, the physical parameter in the eqations is the ratio of the two velocities which characterize the situation. So at $\Gamma/(2\pi) = v_\infty$ the air is being rotated as rapidly to the left (on the cylinder bottom) as it is moving rightward by the cylinders translation. At $\Lambda = 2$ the air at $\theta = 3\pi/2$ is moving forward at the same speed as the cylinder, Eq. (42). For larger Λ there is no stagnation point on the cylinder surface.

2. Bernoulli lift and Magnus lift. The situation described in class, called Bernoulli lift above, is different from the the case of the **Magnus** force. See the accompanying figures and note carefully the qualitative difference between the two situations.

1. Magnus lift (force). The ball (or cylinder) moves through the air with translational velocity v_0. The **ball** is rotated so the one side of it moves more (less) rapidly with respect to the air than the other. See Fig. 1. There is a RHR (Right Hand Rule) for finding the direction of the force on the ball in terms of the direction of translational

motion and the vector direction of the rotation (also found with a RHR). [A Bernoulli argument applied to this case usually gives the wrong answer.]

2. Bernoulli lift. The ball (or cylinder) moves through the air with translational velocity v_0. The **air** is rotated so the air on one side of the ball moves more (less) rapidly with respect to the ball than the air on the other side. See Fig. 2. There is a RHR (Right Hand Rule) for finding the direction of the force on the ball in terms of the direction of translational motion and the vector direction of the rotation of the air (also found with a RHR).

3. Nostrum. [From Merriam-Webster: nostrum = ' a "medicine" of secret composition recommended by its preparer but usually without scientific proof... '.] The problem we have just worked on, motion of a cylinder in stirred air, comes with lots of analytic action: from $F(z)$ to ϕ and ψ to equations for the stream lines, etc. Some of the results are reasonably elaborate. One can often come to an understanding of the content of an analytic but opaque result by employing numerical methods in parallel with analytic methods. The idea is not to throw the problem at a computer but to complement analytic results, e.g., see what an equation says, with judicious numerical activity. Mathematica, Matlab, Maple, Mathcad, other? are very helpful in this context. I strongly recommend that you try to acquire a facility with numerical work that parallels the facility you expect of yourself with analytic work.

Jackson asks for solution to a problem in the form

$$A_\phi(\rho, z) = \frac{4Ia}{c} \int_0^\infty dk \;\cos kz I_1(k\rho_<) K_1(k\rho_>). \tag{2}$$

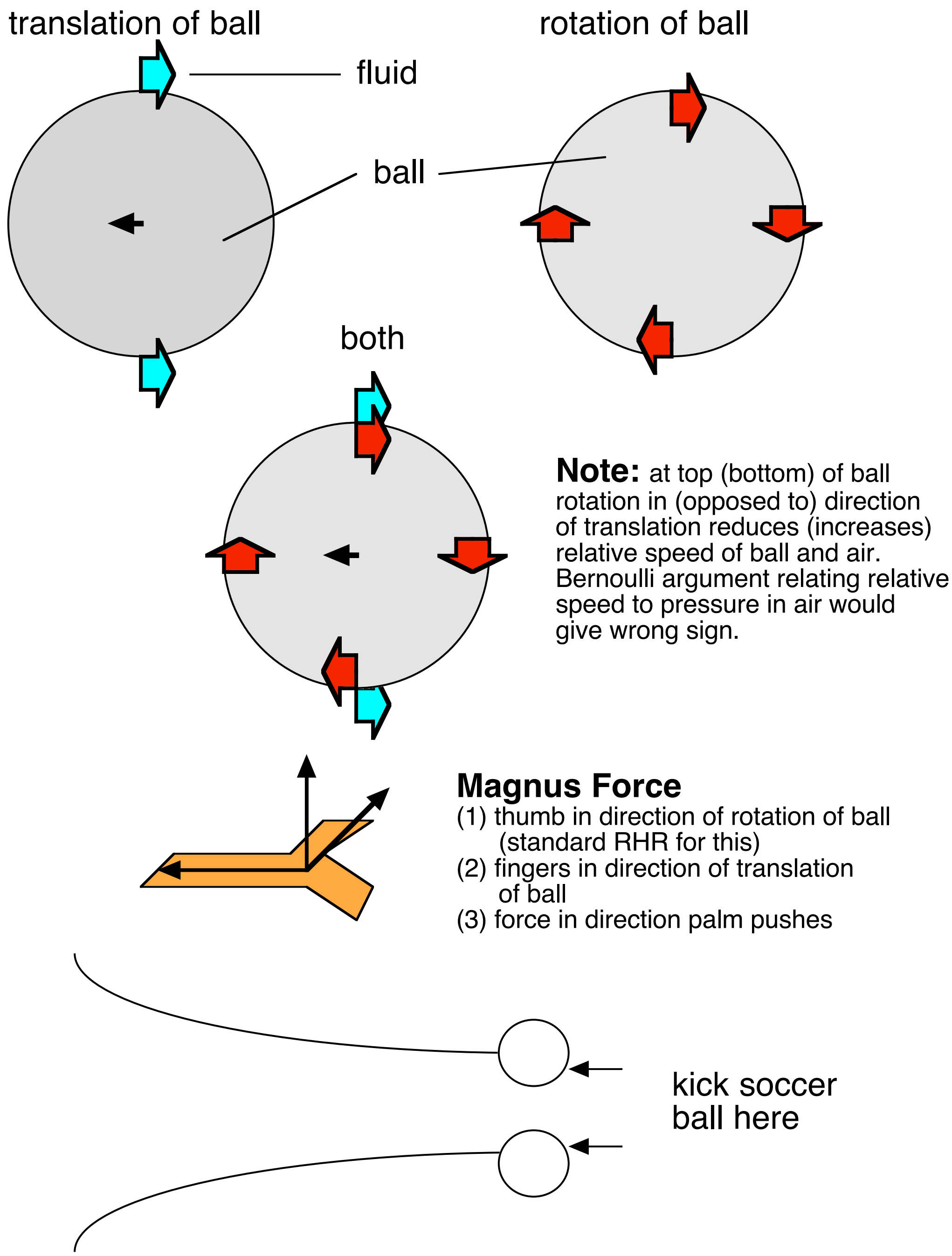

FIG. 1: Magnus Force.

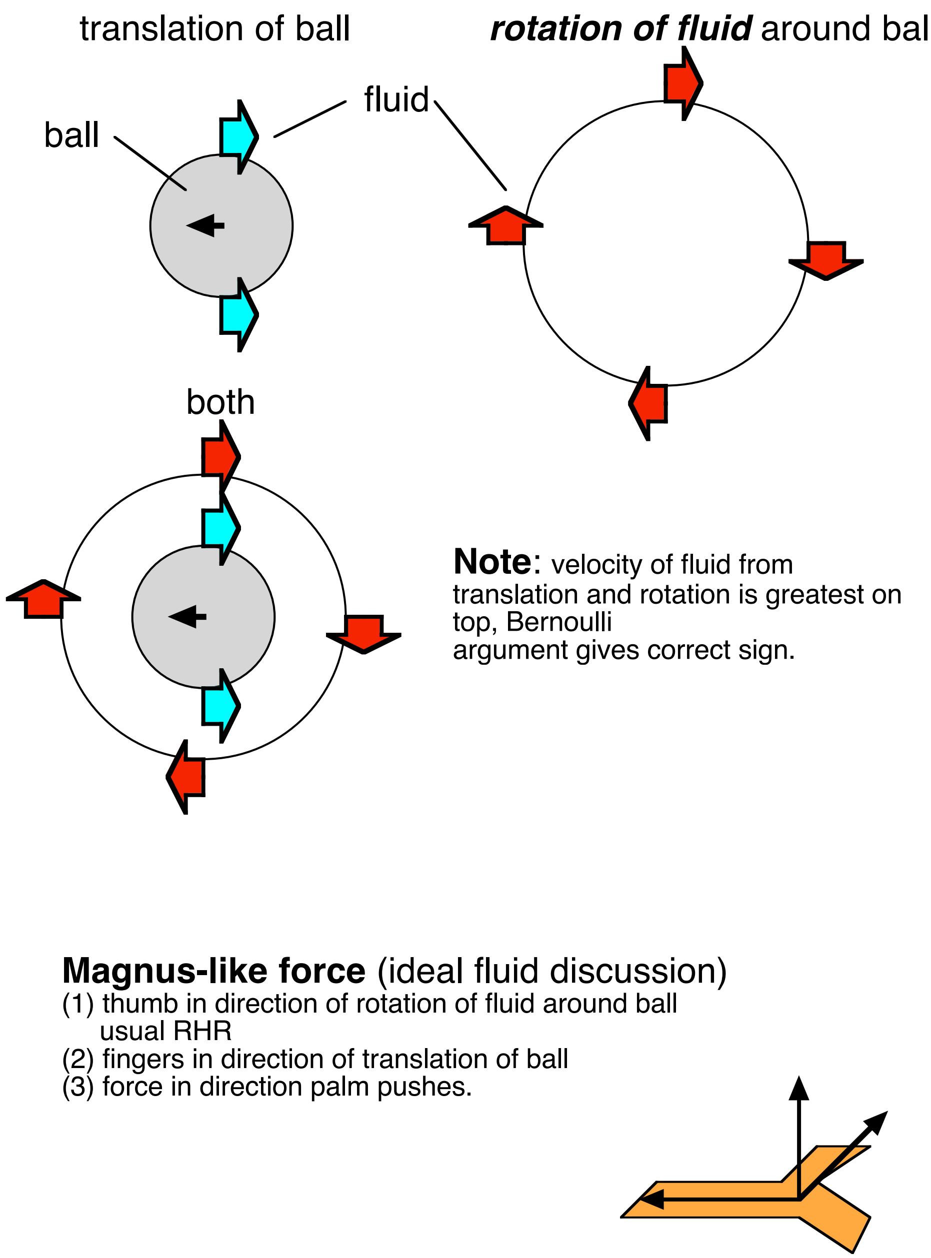

FIG. 2: Bernoulli Lift.